DERIVATION OF THE LEHNERT FIELD EQUATIONS FROM GAUGE THEORY IN THE VACUUM: SPACE CHARGE AND CURRENT

ABSTRACT

It is shown that the Lehnert field equations in the vacuum, with concomitant space charge and current, can be derived straightforwardly from standard gauge theory applied in the vacuum, using the concept of covariant derivative and Feynman's universal influence. The Lehnert and Proca field equations are shown to be inter-related through the well known de Broglie Theorem, in which the photon mass can be interpreted as finite. These ideas go some way towards addressing the inconsistency inherent in Maxwell's famous displacement current, which has no concomitant vacuum space charge.

INTRODUCTION

It is well known that contemporary gauge field theory uses the covariant derivative under all circumstances {1, 3}, including source free regions in the vacuum. If the covariant derivative is not used, special relativity is violated. In this note, it is shown that a simple application of gauge field theory to the electromagnetic field in the vacuum, using complex conjugate covariant derivatives, leads to the form of the Lehnert field equations {4-6}, which have advantages over the Maxwell-Heaviside field equations in which a covariant derivative is not used in the vacuum. The Lehnert field equations provide longitudinal solutions in vacuo and are consistent with finite photon mass. In section 2 of this paper, they are derived from vacuum gauge theory with covariant derivatives, and in Section 3, related to the Proca equation through the de Broglie Theorem. A gauge invariant form of the Proca equation is suggested by this analysis. Therefore the Lehnert equations in the vacuum are transitional between the Maxwell-Heaviside equations and the O(3) electrodynamical equations which define the Evans-Vigier field $\boldsymbol{B}^{(3)}$ {7-12}. A novel interpretation due to Crowell is suggested for the meaning of the vacuum space charge.

GAUGE STRUCTURE OF THE LEHNERT FIELD EQUATIONS.

The Lehnert field equations in the vacuum (source free region) are U(1) symmetry field equations in which the covariant derivatives are complex conjugates:

$$\left(\partial^{\upsilon} + i\frac{e}{\hbar}A^{\upsilon}\right)\tilde{F}_{\mu\upsilon} = 0 \tag{1}$$

$$\left(\partial^{\upsilon} - i\frac{e}{\hbar}A^{\upsilon *}\right)F_{\mu\upsilon} = 0 \tag{2}$$

Here $F_{\mu\upsilon}$ is the U(1) field tensor in the vacuum and $\tilde{F}^{\mu\upsilon}$ its dual. In using the covariant derivatives in the vacuum, the ratio $e/\hbar$ is considered to be a fundamental constant: the ratio of the elementary charge e to the Dirac constant $\hbar$. The vector potential A^{υ} is complex and $A^{\upsilon *}$ is the complex conjugate of A^{υ}. The vacuum electromagnetic field equations are therefore written with covariant derivatives, because any other procedure violates special relativity {1-3} and contemporary gauge theory.

Equation (2) can be written as:

$$\partial^{\upsilon} F_{\mu\upsilon} = i\frac{e}{\hbar} A^{\upsilon *} F_{\mu\upsilon} . \tag{3}$$

The divergence of the electric field in the vacuum is therefore non-zero, as postulated initially by Lehnert and developed by Lehnert and Roy {6}:

$$\nabla \bullet \boldsymbol{E} = -i\frac{e}{\hbar} \boldsymbol{A}^{*} \bullet \boldsymbol{E} \equiv \frac{\rho}{\varepsilon_0}. \tag{4}$$

If we accept plane wave solutions as a first approximation, the divergence becomes:

$$\nabla \bullet \boldsymbol{E} = -\frac{e}{\hbar} A^{(0)} E^{(0)} \tag{5}$$

which has been identified with the charge density ρ introduced by Lehnert {4-6} in the vacuum and developed extensively by Lehnert and Roy {6}. Therefore, it has been shown that the space charge originates in contemporary gauge field theory through the rigorous use of the covariant derivative in the vacuum.

Similarly, eqn. (2) shows that the current in the vacuum introduced by Lehnert into the vacuum Ampère-Maxwell equation is:

$$\boldsymbol{J} = -ic\varepsilon_0 \frac{e}{\hbar} (A^{0*} \boldsymbol{E} + \boldsymbol{A}^{*} \times \boldsymbol{E}). \tag{6}$$

If we use plane waves for $\boldsymbol{A}$ and $\boldsymbol{E}$, this reduces to:

$$\mathrm{Re}\,\boldsymbol{J} = -ic\varepsilon_0 \frac{e}{\hbar} A^{0*}\,\mathrm{Im}\,\boldsymbol{E}. \tag{7}$$

We have the results, in the plane wave approximation:

$$\nabla \bullet \boldsymbol{E} = 0 \tag{8}$$

$$\nabla \times \boldsymbol{E} \neq \boldsymbol{0} \tag{9}$$

which are consistent with the results on p. (9), eqns. (2.3) and (2.11) of Lehnert and Roy {6}:

$$\boldsymbol{j} = \tilde{\rho} \boldsymbol{C} \tag{10}$$

$$\nabla \bullet \boldsymbol{C} = 0 \tag{11}$$

$$\nabla \times \boldsymbol{C} \neq 0 \tag{12}$$

where C is a velocity. The Lehnert equations take the form of the Proca equation, which implies a non-zero mass of the photon. It is shown in the next section that eqn. (3) can be related to the Proca equation through the de Broglie Theorem.

DERIVATION OF THE DE BROGLIE THEOREM FROM VACUUM GAUGE FIELD THEORY.

The Proca equation is {7-12}:

$$\partial^{\upsilon} F_{\mu\upsilon} = -\left(\frac{m_0 c}{\hbar}\right)^2 A_{\mu} \tag{13}$$

where m_0 is the rest mass of the photon in S.I. units. In structure, it is similar to eqn. (3). However eqn. (3) is gauge covariant whereas the Proca equation is not. This is a well known problem with the latter equation, which was developed in the thirties of this century. This suggests that eqn. (3), the Lehnert equation essentially, could be a gauge covariant form of the Proca equation. Contemporary gauge theory {1-3} asserts that for electrodynamics, the constant $e/\hbar$ is always present in the vacuum as a fundamental constant. Once this is realized, Lehnert's space charge and current appear in the vacuum and an inconsistency inherent in Maxwell's use of a vacuum displacement current without a vacuum space charge is addressed through a gauge theoretical structure equivalent to the Lehnert field equations in the vacuum.

If we set the index $\mu = 0$ in equations (3) and (13), we find that:

$$\nabla \bullet \boldsymbol{E} = -\left(\frac{m_0 c}{\hbar}\right)^2 A_0 = -\frac{e}{\hbar} A^{(0)} E^{(0)} \tag{14}$$

and from units:

$$A_0 = c A^{(0)}. \tag{15}$$

Therefore there exists a vacuum Lorentz force magnitude:

$$F = eE^{(0)} = \frac{m_0^2 c^3}{\hbar}. \tag{16}$$

Now use the relation {7-12} from gauge theory with covariant derivatives:

$$\hbar \kappa = e A^{(0)} \tag{17}$$

and the vacuum relations:

$$A^{(0)} = \frac{B^{(0)}}{\kappa} = \frac{E^{(0)}}{c\kappa} \tag{18}$$

where κ is a wavenumber, then

$$\hbar \omega = m_0 c^2 \tag{19}$$

which is the de Broglie Theorem {7-12}. Therefore eqns. (3) and (13) are consistent provided the de Broglie Theorem is accepted.

The latter can be interpreted with m_0c^2 as a "pure energy", or may be interpreted as indicating a non-zero photon mass m_0, in which case it is well known {7-12} that the photon becomes a relativistic particle and that the constant c is no longer interpreted as the speed of light.

Finally, we have the additional relation for the momentum of the photon:

$$p = eA^{(0)} = m_0 c = \hbar\kappa \tag{20}$$

which again relies on the existence of the elementary charge e in the vacuum. Here again, m_0c can be interpreted as a "pure momentum" or as the rest momentum of a photon with finite mass.

DISCUSSION

We note from eqn. (1) that the Gauss and Faraday Laws are the same as those used by Lehnert, so there is no magnetic monopole in this theory. However, the existence of e in vacuo makes a profound difference to quantum electrodynamics, especially if the latter is developed in an O(3) gauge theory {7-12}. One possible interpretation introduced by Crowell is that the Lehnert current is topologically induced by charges which do not interact with the electromagnetic field, giving an effect similar to the Aharonov-Bohm effect. The next level of development, O(3) electrodynamics {7-12}, uses an O(3) internal symmetry to define a longitudinal magnetic field in the vacuum:

$$\boldsymbol{B}^{(3)*} = -i\frac{e}{\hbar}\boldsymbol{A} \times \boldsymbol{A}^* \tag{21}$$

one which again relies on the existence in the vacuum of the constant $e/\hbar$. As argued elsewhere, {7-12} this does not lead to a charged photon, and is consistent with the empirical existence of circular polarization, characterized by the third Stokes parameter {12} proportional to $\boldsymbol{A} \times \boldsymbol{A}^*$. This level of electrodynamics ("O(3) electrodynamics") has been extensively developed {7-12} and is in fact implied by the Lehnert electrodynamics because of the use of the complex conjugate potentials in the Lehnert electrodynamics. The latter is therefore an important transitional step between Maxwell-Heaviside and O(3) electrodynamics, which has a variety of novel properties, one of the most important of which is the existence of longitudinal field components in the vacuum.

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