APPENDICES

APPENDIX ONE: SYMMETRY APPLIED TO THE MAXWELL-HEAVISIDE FIELD EQUATIONS

ABSTRACT

The Maxwell-Heaviside Laws in S.I. units are as follows:

$$\nabla \cdot \boldsymbol{D} = \rho; \qquad \nabla \cdot \boldsymbol{B} = 0;$$
$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}; \qquad \nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \tag{1}$$

where $\boldsymbol{D}$ is electric displacement, ρ is charge density, $\boldsymbol{B}$ is magnetic flux density; $\boldsymbol{E}$ is electric field strength, $\boldsymbol{H}$ is magnetic field strength, $\boldsymbol{J}$ is current density. The Lorentz force law is:

$$\boldsymbol{F} = \rho \boldsymbol{E} + \boldsymbol{J} \times \boldsymbol{B}. \tag{2}$$

These laws of classical electrodynamics are invariant under the following fundamental symmetry operations:

$$\begin{aligned} &1) \text{ parity inversion symmetry } (P), (X,Y,Z) \to (-X,-Y,-Z). \\ &2) \text{ motion reversal symmetry } (T), t \to -t \\ &3) \text{ charge conjugation symmetry } (C), e \to -e. \end{aligned} \tag{3}$$

Some examples of these symmetries are given below, where $\boldsymbol{r}$ is the classical position vector, $\boldsymbol{v}$ is the classical linear velocity vector, and is $\boldsymbol{J}$ the classical orbital angular momentum vector:

$$\begin{aligned} &P(\boldsymbol{r}) = -\boldsymbol{r}; && P(\boldsymbol{v}) = -\boldsymbol{v}; && P(\boldsymbol{J}) = \boldsymbol{J}; \\ &T(\boldsymbol{r}) = \boldsymbol{r}; && T(\boldsymbol{v}) = -\boldsymbol{v}; && T(\boldsymbol{J}) = -\boldsymbol{J}; \\ &P(\boldsymbol{E}) = -\boldsymbol{E}; && P(\boldsymbol{B}) = \boldsymbol{B}; && P(\nabla) = -\nabla; \\ &T(\boldsymbol{E}) = \boldsymbol{E}; && T(\boldsymbol{B}) = -\boldsymbol{B}; && T(t) = -t. \end{aligned} \tag{4}$$

Applying (4) to (1) leaves the received classical laws of electrodynamics unchanged, and so leaves the solutions to these equations unchanged. This is true in general in classical physics. It follows that the classical Maxwell-Heaviside electromagnetic phase has the following basic properties:

$$\begin{aligned} P(\omega t - \kappa \cdot \boldsymbol{r}) &= \omega t - \kappa \cdot \boldsymbol{r} \\ T(\omega t - \kappa \cdot \boldsymbol{r}) &= \omega t - \kappa \cdot \boldsymbol{r} \\ C(\omega t - \kappa \cdot \boldsymbol{r}) &= \omega t - \kappa \cdot \boldsymbol{r} \end{aligned} \tag{5}$$

a result which is consistent with the fact that the received Maxwell-Heaviside phase is a unitless number, a scalar invariant under P, T and C.

It also follows that the d'Alembert wave equation in vacuo is invariant under P, T and C:

$$\begin{aligned} P(\Box A^{\mu} = 0) &= (\Box A^{\mu} = 0) \\ T(\Box A^{\mu} = 0) &= (\Box A^{\mu} = 0) \\ C(\Box A^{\mu} = 0) &= (\Box A^{\mu} = 0) \end{aligned} \tag{6}$$

The individual symmetries are:

$$\begin{array}{lll} P(\Box) = \Box; & T(\Box) = \Box; & C(\Box) = \Box \\ P(A^{\mu}) = A^{\mu}; & T(A^{\mu}) = A^{\mu}; & C(A^{\mu}) = -A^{\mu} \end{array} \tag{7}$$

which follow from the units of the four potential:

$$A^{\mu} = (\phi, cA) \quad J/C. \tag{8}$$

These received equations of classical electrodynamics give us no further information.

It is possible to illustrate catastrophic failures of this view of classical electrodynamics by the use of symmetry. Examples are given as follows.

1) The Sagnac Effect

This is well described in the papers of the volume. With platform at rest:

$$T(A) = C \tag{9}$$

where A is the anticlockwise motion of a beam of light and C is its clockwise motion. The Maxwell-Heaviside phase is exactly the same for each journey because the phase is invariant under T. There is no phase change and no interference, contrary to observation, with platform at rest. In other words:

$$(\kappa \cdot \boldsymbol{r})_A = (\kappa \cdot \boldsymbol{r})_C \tag{10}$$

and there is no way in which the Maxwell-Heaviside theory can explain the observed phase change in the Sagnac effect with platform at rest. The theory is Lorentz frame invariant in vacuo and there is no effect of rotation on the Maxwell-Heaviside equations. They cannot describe what is usually known as the Sagnac effect, the extra phase shift produced by a rotating platform. These are major failures of the received view.

2) Michelson Interferometry and Normal Reflection

These are examples of parity inversion, i.e., $Z \rightarrow -Z$, and the Maxwell-Heaviside phase is unchanged by P. Therefore

$$P(\kappa \cdot \boldsymbol{r}) = \kappa \cdot \boldsymbol{r}. \tag{11}$$

In Michelson interferometry, there is no phase change, contrary to observation, and normal reflection, as usually described by:

$$\omega t - \kappa \cdot \boldsymbol{r} \xrightarrow{?} \omega t + \kappa \cdot \boldsymbol{r} \tag{12}$$

violates P symmetry. These are two more major failures of Maxwell-Heaviside theory. Since these failures are revealed by basic symmetry arguments, they are irreparable within the context of the Maxwell-Heaviside theory. Normal reflection is a special case of one of the oldest laws in physics, Snell's law, and the implication is that Snell's Law is not described by the Maxwell-Heaviside Theory. One counterexample is enough to refute a theory, and we have given several counter examples.

O(3) ELECTRODYNAMICS

As in several papers of the collection, the Sagnac effect is described by:

$$\oint_C \kappa \cdot d\boldsymbol{r} - \oint_A \kappa \cdot d\boldsymbol{r} = g \oint_S \boldsymbol{B}^{(3)} \cdot d\boldsymbol{Ar} \tag{13}$$

and Michelson interferometry and normal reflection is described by:

$$\oint \kappa \cdot d\boldsymbol{r} = g \oint_S \boldsymbol{B}^{(3)} \cdot d\boldsymbol{Ar}. \tag{14}$$

The P and T symmetry of both sides is in each case negative. So O(3) electrodynamics makes sense, normal reflection is described by a P negative equation; the Sagnac effect by a T negative equation. The extra phase change in the Sagnac effect is an O(3) gauge transform, a physical rotation of the platform.

Therefore we reach the important and far reaching conclusion that all interferometry and all physical optics is an O(3) gauge theory. Any other conclusion is self-inconsistent, or contra-indicated by empirical data and symmetry. It is not valid to use a U(1) theory in one situation and an O(3) theory in another. The former is incorrect, so the latter must be developed for all situations in electrodynamics and optics.