AN EXPLANATION OF SNELL'S LAW USING NON-ABELIAN ELECTRODYNAMICS

ABSTRACT

The conventional explanation for simple reflection and Snell's law in Maxwell-Heaviside theory violates the parity inversion operator P. It is shown that a self-consistent explanation is given in non-Abelian electrodynamics, using the appropriate Stokes theorem for the electromagnetic phase.

INTRODUCTION

Snell's Law and normal reflection in optics are of course well verified empirically, but the Maxwell-Heaviside explanation {1} of this simple phenomenon of optics is shown to violate parity inversion symmetry (P). It is suggested that a non-Abelian electromagnetic phase, based on the appropriate Stokes theorem {2}, gives a self-consistent description of the effect, and equates the dynamical and topological phases. The topological phase is missing entirely from the Maxwell-Heaviside theory, but is well observed {3-5} in interferometry.

P VIOLATION

The Maxwell-Heaviside theory of electrodynamics is a U(1) gauge theory {6, 7}, so is referred to, for brevity, simply as "U(1)". Reflection is described in terms of the incident U(1) phase, and the reflected U(1) phase. The former is

$$i(\kappa \cdot \boldsymbol{r} - \omega t) \tag{1}$$

and the latter for normal reflection is

$$i(-\kappa \cdot \boldsymbol{r} - \omega t) \tag{2}$$

where $\boldsymbol{\kappa}$ is the wave-vector at position $\boldsymbol{r}$, and ω the frequency at instant t. Normal reflection is equivalent to parity inversion, P, whose effect is:

$$P(X,Y,Z) = -X,-Y,-Z \tag{3}$$

if r is in the Z axis then $P(Z) = -Z$ by definition, and $P(r) = -r$. However, $P(\boldsymbol{\kappa}) = -\boldsymbol{\kappa}$ and $P(\omega\ t) = \omega\ t$. Therefore:

$$P(\kappa \cdot \boldsymbol{r} - \omega t) = \kappa \cdot \boldsymbol{r} - \omega t \tag{4}$$

and the usual U(1) description of normal reflection violates parity inversion symmetry and is invalid. This is easily seen from the fact that the U(1) phase is a number invariant under P, and motion reversal T. Therefore U(1) is unable to describe Sagnac and Michelson interferometry, as has been recently realized {8-11}.

SELF-CONSISTENT DESCRIPTION OF SNELL'S LAW IN NON-ABELIAN ELECTRODYNAMICS.

If electrodynamics is described by an O(3) gauge theory, {12-15} the Snell Law originates in a phase which is described by a non-Abelian Stokes theorem.{16} It has been shown elsewhere {10} that this can be reduced to:

$$\oint \kappa \cdot d\boldsymbol{Z} = g \int \boldsymbol{B}^{(3)} \cdot d\boldsymbol{S} \qquad (5)$$

where the left hand side is a line integral, and the right hand side is an area integral over the Evans-Vigier field $B^{(3)}${8-11}. If a beam of light originates at O and is normally reflected from a perfectly reflecting mirror at the point Z, the line integral is:

$$\oint \kappa \cdot d\boldsymbol{Z} = \int_0^Z \kappa\, dZ - \int_Z^0 \kappa\, dZ = 2\kappa Z. \qquad (6)$$

Notice that this gives, by chance, the same phase change, $2\kappa Z$ as in the U(1) description of normal reflection, which is therefore fortuitously useful as a calculating device, but physically incorrect.

The area integral on the right hand side of eqn. (5) is a topological phase {3-5}. Using the definition $g = \kappa/A^{(0)}$ and

$$B^{(3)} = \kappa A^{(0)} \qquad (7)$$

the right hand side becomes $\kappa^2 S$, where S is an area:

$$S = \frac{2Z}{\kappa}. \qquad (8)$$

If the distance OZ is n wavelengths, λ, then the area is:

$$S = \frac{n\lambda^2}{\pi} \qquad (9)$$

Normal reflection is a special case of the Snell Law, and the latter can always be reduced to normal reflection by utilizing the components in Z of the incident and reflected beams. These components are equal and opposite as in eqn. (6) because the incident and reflected angles to the normal are equal by Snell's Law.

DISCUSSION

The outcome of this very simple example is that all of electrodynamics (classical and quantum) must be upgraded to a gauge theory of higher symmetry than U(1). The O(3) gauge symmetry proves to be effective {8-11}. The Snell Law is of course empirically well verified, but the U(1) description of it violates P. The O(3) description in eqn. (5) is self-consistent, because under P, both sides are negative. The left hand side is negative because the line integral changes sign under P, and the right hand side is negative because the integral is negative under P (product of an axial vector $B^{(3)}$ and a polar vector S). A rigorous derivation of eqn. (5) is available elsewhere {10}.

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{16} A transparent form of this theorem is given by Broda in ref. (2).