PREFACE

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ABSTRACT

The great Maxwell-Heaviside theory of electrodynamics was initiated by J.C. Maxwell [1-3] in his attempts to explain Faraday's electrotonic state [4]. The latter is now known as the vector potential, a term introduced by Maxwell himself, who regarded the potential as physical, and who regarded charge as being the result of the field. Later, Heaviside [5]and his contemporaries modified the original intent of Maxwell by asserting that the potential is a mathematical device of no physical significance. This led to the introduction of gauge freedom and the great gauge field theories which have dominated the later twentieth century in natural philosophy. The original equations and intent of J.C. Maxwell were quite different, and his original equations [1-3] are written in integral form with no approximations. The original concept by Maxwell that charge be the result of the field was later reversed by Lorentz, who regarded the field as the result of charge, which is the source of energy. This is clearly in violation of conservation of both energy and momentum. The usefulness of the classical theory of electrodynamics needs not be emphasized, there are millions of papers on the subject and thousands of textbooks. However, it is riddled with flaws, and in this collection of papers, some of these are revealed, with suggestions for a new type of electrodynamics based on contemporary gauge theory.

For engineers not familiar with the tensorial form of the Maxwell-Heaviside equations introduced by Lorentz and Poincaré, and independently by Einstein, it is useful to demonstrate the approximations inherent in the Maxwell-Heaviside theory by considering the familiar vector equations in the vacuum, for clarity and simplicity of demonstration. It is sufficient to consider only the Faraday Law and the Ampère-Maxwell Law, which is the Ampère Law modified by Maxwell's displacement current. The latter gives rise to electromagnetic transmission through the vacuum and the usefulness of this phenomenon is obvious. Therefore the Maxwell-Heaviside and Lorentz equations are great laws of physics, which happen to be erroneous.

The Faraday Law of induction is:

$$\nabla \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = \mathbf{0} \tag{1}$$

where $\boldsymbol{E}$ is electric field strength (volts per meter) and $\boldsymbol{B}$ is magnetic flux density in Weber per square meter. The Ampère-Maxwell Law in vacuo is:

$$\nabla \times \boldsymbol{B} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} = \mathbf{0} \tag{2}$$

where c is a universal constant of special relativity, the speed of light in vacuo in meters/second.

These are the differential forms of Maxwell's original integral equations [1-3] which he formulated after years of effort. If it is asserted that the electromagnetic entity is made up of fields, these equations must be solved along with the vacuum Gauss Law

$$\nabla \cdot \boldsymbol{B} = 0 \tag{3}$$

and Coulomb Law in the vacuum:

$$\nabla \cdot E = 0 \tag{4}$$

It was neither Faraday's nor Maxwell's intent to describe the electromagnetic entity in terms of fields, their description rested on a physical potential, described by Faraday as the electrotonic state. Schwarzschild and Whittaker have shown that this is indeed the case, that there is no need for fields to describe the electromagnetic entity in vacuo. Some of Whittaker's work in this direction is described in this collection.

Heaviside [5] greatly confused the subject through the following procedure, well described in thousands of libraries. Firstly, ***B*** and ***E*** are described in terms of a scalar potential ϕ and a vector potential ***A***, which can be combined to give a four vector of special relativity:

$$A^{\mu} = (\phi, cA) \tag{5a}$$

$$\boldsymbol{B} = \nabla \times A \tag{5b}$$

$$\boldsymbol{E} = -\frac{\partial A}{\partial t} - \nabla \phi \tag{5c}$$

This procedure produces:

$$-\nabla \times \left(\frac{\partial \mathrm{A}}{\partial t} + \nabla \phi \right) + \frac{\partial}{\partial t}(\nabla \times A) = \mathbf{0} \tag{6}$$

which is correct without approximation, but also produces the equation:

$$\nabla \left(\nabla \cdot A + \frac{1}{c^2} \frac{\partial \phi}{\partial t} \right) + \nabla^2 A - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} A = \mathbf{0} \tag{7}$$

which was unraveled to give the Lorenz condition:

$$\nabla \cdot A + \frac{1}{c^2} \frac{\partial \phi}{\partial t} = 0 \tag{8}$$

and the vacuum d'Alembert equation:

$$\nabla^2 A - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} A = \mathbf{0} \tag{9}$$

This is an arbitrary procedure which is based on the assertion that there is gauge freedom arising from eqns. (5b) and (5c). If ***A*** is replaced by ***A*** - ∇x in eqn. (5b), where x is an arbitrary function, then ***B*** is unchanged. Similarly if ϕ is replaced by $\phi + \partial x/\partial t$ in eqn. (5c), then ***E*** is unchanged. Heaviside's contribution was to assert that ***A*** and ϕ are therefore defined only up to arbitrary variables ∇x and $\partial x/\partial t$ and therefore cannot have physical meaning. Only the fields ***B*** and ***E*** are physical, according to Heaviside. This assertion is quite false, and is refuted in this collection of papers through reference to Whittaker's work [6, 7] and to a new approach to electrodynamics based on contemporary gauge theory.

The so-called gauge freedom in electrodynamics allows the Lorenz condition to be replaced by the radiation gauge condition:

$$\nabla \cdot A = 0 \tag{10}$$

in which ϕ is zero. So there arise different gauges in electrodynamics, the radiation and Lorenz gauges are two examples. The overall effect of this concept of gauge freedom is to add an arbitrary number to the electromagnetic phase under gauge transformation (5b) and (5c) in the Maxwell-Heaviside theory. Since an arbitrary number is unphysical, the concept of gauge freedom is meaningless, and the elaborate mathematical developments thereof are of no interest to physics. Barrett, [8] for example, has shown repeatedly that the potential in electrodynamics is physical, as was obvious to both Faraday and Maxwell. In this collection of papers, it is shown conclusively that there are serious elementary flaws in the Maxwell-Heaviside theory, for example, its failure to describe reflection, and interferometry.

Four papers in this issue demonstrate that there is no gauge freedom in Maxwell-Heaviside theory; these are papers which develop Whittaker's original expression [6, 7] of all electric and magnetic fields in terms of two longitudinal flux densities g and f, with units of Weber. Under well defined conditions, it is shown from Whittaker's work that the only possibility is $x = 0$. There can be potentials without fields, and under a well defined condition, the only potential present in the vacuum is a physical scalar potential which is structured, and can be quantized to the photon straightforwardly. In this condition, there is no vector potential and no fields. The only other entities present are g and f in Maxwell-Heaviside theory. The magnetic flux density corresponding to g and f is also longitudinal in the vacuum, and is denoted by the symbol $\boldsymbol{B}^{(3)}$. This is however not given by Maxwell-Heaviside theory, but rather by a gauge field theory of different symmetry. This theory is the core of the novel electrodynamics presented in this collection, and it is denoted "O(3) electrodynamics". The fundamental reason for this is that there exist cyclic relations in the vacuum between $\boldsymbol{B}^{(3)}$, which is phaseless, and the transverse plane waves $\boldsymbol{B}^{(1)} = \boldsymbol{B}^{(2)*}$ in a complex basis for space denoted ((1), (2), (3)) [9-12]. The relation is cyclically symmetric and is mathematically classified as "non-Abelian". It has the symmetry of the rotation group in three dimensional space, O(3). Gauge freedom in the Maxwell-Heaviside theory, on the other hand, is classified as Abelian and described by the rotation group O(2) in a plane. Gauge freedom is however non-existent, as shown by the four papers in this collection devoted to Whittaker's work, and so the O(2) gauge theory must be discarded. This is classified mathematically as Abelian, because it is linear in nature; products such as $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$ do not exist in O(2), and neither does $\boldsymbol{B}^{(3)}$. The papers describe many cases which show that $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$ exists physically, as in magneto-optics for example.

The most concise form of the new field equations is as follows:

$$D_\mu \tilde{\boldsymbol{G}}^{\mu\nu} = \boldsymbol{0} \tag{11}$$

$$D_\mu \boldsymbol{H}^{\mu\nu} = \boldsymbol{J}^\nu . \tag{12}$$

The meaning of these equations is developed in the papers, and they are also broken out into vector form in the papers. So without going into superfluous detail in this preface, suffice it to say that these equations have many more solutions than the Maxwell-Heaviside equations and are more self-consistent. They are able to describe phenomena such as reflection and interferometry that the Maxwell-Heaviside theory is unable to describe self-consistently. They predict, in essence, that there is an internal structure to the Maxwell-Heaviside theory which, if engineered, may solve many energy problems. They can be reduced (in a particular solution) to Maxwell-Heaviside type equations for transverse plane waves, together with equations for the

$B^{(3)}$ field, defined by $B^{(3)} = B^{(0)}k$, where is a unit vector in the propagation axis. The $B^{(3)}$ equations in the vacuum are therefore:

$$\nabla \cdot \boldsymbol{B}^{(3)} = 0 \tag{13}$$

$$\nabla \times \boldsymbol{B}^{(3)} = \mathbf{0} \tag{14}$$

$$\frac{\partial \boldsymbol{B}^{(3)}}{\partial t} = \mathbf{0} \tag{15}$$

and imply that $B^{(3)}$ is a self-dual field, conceptually unknown in Maxwell-Heaviside theory. The structure of eqns. (11) and (12) is that of a theory introduced to generalize electrodynamics in 1954 by Yang and Mills, [13] but with the key difference that the internal gauge space is **physical**.

This collection of papers suggests numerous flaws in the Maxwell-Heaviside theory and how the new O(3) theory corrects them. Some of these are given below:

1. There is self-inconsistency in the gauge theory that leads to Maxwell-Heaviside theory in that the former eliminates a commutator such as $B^{(1)} \times B^{(2)}$ by definition. This commutator is however an observable of the inverse Faraday effect, and is put in phenomenologically in the Maxwell-Heaviside theory to explain the effect. In O(3) electrodynamics, this commutator is defined self-consistently and is proportional to the $B^{(3)}$ field and to the third Stokes parameter.

2. The Sagnac effect with platform at rest cannot be described by Maxwell-Heaviside theory due to motion reversal symmetry. There is no phase shift, contrary to observation. The O(3) electrodynamics succeeds in explaining the phase shift with platform at rest and in motion to great precision.

3. The phase shift of the Michelson interferometer cannot be described by Maxwell-Heaviside theory, which gives a null result, contrary to observation, but in O(3) electrodynamics, it is described by a non-Abelian Stokes theorem involving the $B^{(3)}$ field. There is a topological phase equivalent to the dynamical phase, as observed empirically [14].

4. Normal reflection is not described by the Maxwell-Heaviside theory because of parity inversion symmetry. The O(3) electrodynamics describes the observed phase change precisely through a non Abelian Stokes theorem. This is a general feature of interferometry.

5. The O(3) field equations (11) and (12) reduce under well defined circumstances to Maxwell-Heaviside equations and $B^{(3)}$ equations; and these equations reduce in turn to those of electrostatics, which are well verified empirically.

6. The O(3) equations have been derived independently by Barrett [15] in the homomorphic SU(2) form, and are mathematically correct and self-checking. In this collection and elsewhere, they have been developed extensively in close coordination with empirical data.

7. A careful examination of the Maxwell-Heaviside theory shows that it fails to describe interferometry, while the O(3) equations succeed in giving both the observed dynamical phase and topological phase [14].

8. A consequence of the O(3) theory is an SU(2) × SU(2) electroweak theory, which is given in two papers in this issue, predicting a massive $A^{(3)}$ boson which may be observable on a heavy hadron collider.

9. In this collection, we have developed the work of Whittaker to refute the gauge freedom of the Maxwell-Heaviside theory by counter-example. In the O(3) theory, there is no gauge freedom because it is gauge covariant, not gauge invariant.

10. The self-consistency of the novel O(3) electrodynamics has been tested extensively for self-consistency in the books listed in this special issue, and in the numerous papers also listed.

11. The O(3) electrodynamics have been extended to quantum electrodynamics showing minute but real corrections to the Lamb shift in atomic ^{1}H and to the g factor of the electron. There is scope here for considerable development.

12. No contradiction with empirical data has been found with O(3) electrodynamics although it is not claimed to be a perfect theory. Classical electrodynamics in general is capable of very significant improvement. The Maxwell-Heaviside theory, on the other hand, is now realized to fail catastrophically.

13. The O(3) electrodynamical structure is mathematically that of a Yang-Mills theory with a physical internal gauge space based on the existence of circular polarization, and labeled ((1), (2), (3)).

14. There is a self-inconsistency in the stress energy momentum tensor of the Maxwell-Heaviside theory which is removed by the O(3) theory.

15. The O(3) theory similarly saves the correspondence principle in the Compton effect, through the use of an effective $A^{(3)}$ potential, not to be confused with the massive Crowell boson.

16. The technique of radiatively induced fermion resonance (RFR), which produces fermion resonance without magnets, has been developed as a practical spin-off of O(3) electrodynamics, and promises to introduce a new technology in ESR, NMR and MRI.

17. The O(3) equations (11) and (12) produce soliton and instanton solutions, which are missing conceptually from the Maxwell-Heaviside theory, and conditions under which instantons can be observed in electrodynamics have been defined.

18. The O(3) electrodynamics lead to the Crowell duality principle, the simplest example being the SU(2) × SU(2) electroweak theory producing the observable massive $A^{(3)}$ mentioned already.

19. Covariant O(3) derivatives are used in the O(3) field equations in which the universal constant e has the dual meaning of coupling constant. So quantization of the O(3) theory does not lead to charged photons.

The interested specialist is referred to the text for more details of this emerging subject. There appear to be extensive consequences in electrical engineering because the O(3) equations have so many more solutions than the Maxwell-Heaviside equivalents.

Another important observation is that the Lorenz condition [16] was devised by Ludwig Lorenz of Copenhagen in 1867, not by Henrik Anton Lorentz of Leiden. Therefore it is referred to throughout this collection as the Lorenz condition. It is an arbitrary construct devised for mathematical convenience, and loses considerable information. It is the biggest blunder in classical electrodynamics, and makes canonical quantization based on the Lorenz gauge an arbitrary and meaningless procedure, invalidating for example the Gupta-Bleuler procedure. The Lorenz gauge prohibits the existence of time-like and longitudinal photons, and there is no physical reason for this. Time-like photons can easily be constructed from a structured scalar potential, and spin accorded to the photon through Wigner's method of 1939. Discarding the Lorenz gauge, and the idea of gauge freedom, leads to a considerable amount of new physics. A tiny fraction of what is possible is outlined in this collection. For example, it turns out that O(3) electrodynamics does not incorporate a monopole as material point particle, because it is a theory based on the topology of the vacuum.

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Electric field strength,	$\boldsymbol{E}$	$\mathrm{V\ m^{-1} = J\ C^{-1}\ m^{-1}}$
Electric displacement	$\boldsymbol{D}$	$\mathrm{C\ m^{-2}}$
Magnetic flux density	$\boldsymbol{B}$	$\mathrm{T = Wb\ m^{-2} = J\ s\ C^{-1}\ m^{-2}}$
Magnetic field strength	$\boldsymbol{H}$	$\mathrm{A\ m^{-1} = C\ s^{-1}\ m^{-1}}$
Magnetic vector potential	$\boldsymbol{A}$	$\mathrm{J\ s\ C^{-1}\ m^{-1}}$
Polarization	$\boldsymbol{P}$	$\mathrm{C\ m^{-2}}$
Magnetization	$\boldsymbol{M}$	$\mathrm{A\ m^{-1} = C\ s^{-1}\ m^{-1}}$
Charge density	ρ	$\mathrm{C\ m^{-3}}$
Current density	$\boldsymbol{J}$	$\mathrm{A\ m^{-2} = C\ s^{-1}\ m^{-2}}$
Vacuum permittivity	ϵ_0	$= 8.854188 \times 10^{-12}\ \mathrm{J^{-1}\ C^{-2}\ m^{-1}}$
Vacuum permeability	μ_0	$= 4\pi \times 10^{-7}\ \mathrm{J\ s^2\ C^{-2}\ m^{-1}}$
Electronic *g* factor	g	$= 2.002319314$
Dirac constant	$\hbar$	$= 1.05459 \times 10^{-34}\ \mathrm{J\ s}$
Fine structure constant	α	$= \dfrac{e^2}{4\pi\hbar c\varepsilon_0} = 0.007297351$
Speed of light in vacuo	c	$= 2.997925 \times 10^{8}\ \mathrm{m\ s^{-1}}$
Elementary charge	e	$= 1.60219 \times 10^{-19}\ \mathrm{C}$
Electron mass	m_e	$= 9.10953 \times 10^{-31}\ \mathrm{kg\ m}$
Proton mass	m_p	$= 1.67265 \times 10^{-27}\ \mathrm{kg\ m}$
Bohr magneton	μ_B	$= \dfrac{e\hbar}{2m_e} = 9.27408 \times 10^{-24}\ \mathrm{J\ T^{-1}}$

THE ORIGINAL WHITTAKER PAPERS

Mathematische Annalen, Vol. 57, 1903, p. 333-355.
(Retyped for readability, with corrections)

ON THE PARTIAL DIFFERENTIAL EQUATIONS OF MATHEMATICAL PHYSICS.

E. T. Whittaker in Cambridge.

§ 1.

INTRODUCTION

The object of this paper is the solution of Laplace's potential equation

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=0,$$

and of the general differential equation of wave-motions

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=k^2\frac{\partial^2 V}{\partial t^2},$$

and of other equations derived from these.

In § 2, the general solution of the potential equation is found.

In § 3, a number of results are deduced from this, chiefly relating to particular solutions of the equation, and expansions of the general solution in terms of them.

In § 4, the general solution of the differential equation of wave-motions is given.

In § 5, a number of deductions from this general solution is given, including a theorem to the effect that any solution of this equation can be compounded from simple uniform plane waves, and an undulatory explanation of the propagation of gravitation.

§ 2.

THE GENERAL SOLUTION OF THE POTENTIAL EQUATION.

We shall first consider the equation

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=0,$$

which was originally given by Laplace[1]).

This equation is satisfied by the potential of any distribution of matter which attracts according to the Newtonian Law. We shall first obtain a general form for potential-functions, and then shall shew that this

[1] *Mémoire sur la théorie de l'anneau de Saturne, 1787.*

form constitutes the general solution of Laplace's equation. From the identity

$$\frac{1}{\sqrt{\left\{(x-a)^2+(y-b)^2+(z-c)^2\right\}}}=\frac{1}{2\pi}\int_0^{2\pi}\frac{du}{(z-c)+i(x-a)\cos u+i(y-b)\sin u},$$

we see that the potential at any point (x, y, z) of a particle of mass m, situated at the point (a, b, c), is

$$\frac{m}{2\pi}\int_0^{2\pi}\frac{du}{(z+ix\cos u+iy\sin u)-(c+ia\cos u+ib\sin u)}$$

which, considered as a function of x, y, z, is an expression of the type

$$\int_0^{2\pi} f(z+ix\cos u+iy\sin u,\, u)\,du,$$

where f denotes some function of the two arguments

$$z+ix\cos u+iy\sin u \quad \text{and} \quad u.$$

It follows that the potential of any number of particles $m_1, m_2, \ldots, m_k$, situated at the points $(a_1b_1c_1)$, $(a_2b_2c_2)$, $(a_3b_3c_3)$, . . ., $(a_kb_kc_k)$, is an expression of the type

$$\int_0^{2\pi}\{f_1(z+ix\cos u+iy\sin u,\, u)+f_2(z+ix\cos u+iy\sin u,\, u) \\ +f_k(z+ix\cos u+iy\sin u,\, u)\}du$$

or

$$\int_0^{2\pi} f(z+ix\cos u+iy\sin u,\, u)\,du,$$

where f is a new function of the two arguments

$$z+ix\cos u+iy\sin u \quad \text{and} \quad u.$$

In this way we see that *the potential of any distribution of matter which attracts according to the Newtonian Law can be represented by an expression of the type*

$$\int_0^{2\pi} f(z+ix\cos u+iy\sin u,\, u)\,du.$$

The question now naturally suggests itself, whether the most general solution of Laplace's equation can be represented by an expression of this type. We shall shew that the answer to this is in the affirmative.

For let $V(x, y, z)$ be any solution (single-valued or many-valued) of the equation