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Paper 13

The Charge Quantization Condition: Link Between the O(3) Gauge Group and the Dirac Equation

The charge quantization condition (*CQC*) equates the quantized vacuum photon momentum to the classical product $eA^{(0)}$, where e is the charge on the electron and where $A^{(0)}$ is the scalar magnitude of the potential four-vector of electromagnetic radiation. It is shown that the *CQC* emerges consistently from the expression for the Evans-Vigier field $\boldsymbol{B}^{(3)}$ in the *O(3)* gauge group of vacuum electromagnetism and the Dirac equation for the spinning trajectory of an electron in the field.

Key words: charge quantization condition, $\boldsymbol{B}^{(3)}$ field

13.1 Introduction

The magnetic components of the ordinary plane waves of vacuum electromagnetism are now known [1-10] to act as the source of the magnetizing field $\boldsymbol{B}^{(3)}$, the Evans-Vigier field [6]. The real and physical $\boldsymbol{B}^{(3)}$ field propagates through the vacuum with the plane waves, and is an axial vector directed in the propagation axis. It is an experimental observable, and can be isolated [6,9] from the concomitant plane waves through its magnetization of material matter, in the simplest instance one electron. The magnetization, $\boldsymbol{M}^{(3)}$, is, at microwave frequencies [11], proportional to $I_0^{1/2}$, where I_0 is the power density of the beam in W m^{-2}. Therefore $\boldsymbol{B}^{(3)}$ is a physical magnetic flux density, and is now understood in several different ways [9]. There is no reasonable doubt that it adds a third dimension to the understanding of vacuum electromagnetism.

An immediate consequence is that the gauge group of vacuum electromagnetism can no longer be considered to be the conventional *O(2)* [12], the group of rotations in a plane. The natural generalization to *O(3)*, the group of rotations in three dimensional space, is considered in Sec. 13.2, where it is shown that the field $\boldsymbol{B}^{(3)}$ emerges from *O(3)* gauge geometry as being proportional to the vector product of the plane wave vector potential $A^{(1)}$ with its own complex conjugate $A^{(2)}$. This result leads to the charge quantization condition (*CQC*), which equates the quantized vacuum photon momentum $\hbar\kappa$ to the classical $eA^{(0)}$. Here e is both the charge on the electron and the scaling constant of *O(3)* gauge geometry [12], and $A^{(0)}$ is the scalar magnitude of $A^{(1)}$. In Sec. 13.3, the Dirac equation of one electron in the electromagnetic field is used to produce an expression for $\boldsymbol{B}^{(3)}$ which becomes identical with that derived in Sec. 13.2 by using the *CQC*. The latter therefore makes the *O(3)* gauge group theory of vacuum electromagnetism consistent with the Dirac equation of one electron in the electromagnetic field. Both theories consistently produce $\boldsymbol{B}^{(3)}$ in the vacuum, and a discussion is given of some of the wider implications of the discovery of the Evans-Vigier field.

13.2 The O(3) Symmetry of Vacuum Electromagnetism

The need for an *O(3)* gauge group of vacuum electromagnetism is revealed by the defining Lie algebra of the $\boldsymbol{B}^{(3)}$ field [6],

$$\begin{aligned} \boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} &= iB^{(0)}\boldsymbol{B}^{(3)*}, \\ \boldsymbol{B}^{(2)} \times \boldsymbol{B}^{(3)} &= iB^{(0)}\boldsymbol{B}^{(1)*}, \\ \boldsymbol{B}^{(3)} \times \boldsymbol{B}^{(1)} &= iB^{(0)}\boldsymbol{B}^{(2)*}, \end{aligned} \tag{2.13.1}$$

where $\boldsymbol{B}^{(1)} = \boldsymbol{B}^{(2)*}$ are the magnetic components of the ordinary plane waves. This algebra is *non*-Abelian, compact and semi-simple, and has *O(3)* symmetry [12], not *O(2)*. Therefore the *O(3)* group must be used to describe vacuum electromagnetism in the general theory of gauge geometries [12], a theory which parallels general relativity in its conceptual development. The *O(3)* theory of vacuum electromagnetism is non-Abelian in nature, and therefore the field can act as its own source [6]. Thus, the conjugate product $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$ acts as the source of $\boldsymbol{B}^{(3)}$, a new physical field which propagates through the vacuum with the plane waves, and which is observed through its $I_0^{1/2}$ profile [9]. This inference is reinforced conclusively [9] because the source of $\boldsymbol{B}^{(3)}$ can be described in terms both of a Biot-Savart-Ampère law and as the curl of a vector potential [9]. Therefore $\boldsymbol{B}^{(3)}$ has all the known properties of a magnetic flux density, and acts experimentally as such [6]. In retrospect its existence has already been detected experimentally in second order magneto-optic effects, because the well known conjugate product [13] is $iB^{(0)}\boldsymbol{B}^{(3)*}$, an experimental observable. Here $B^{(0)}$ is the scalar magnitude of $\boldsymbol{B}^{(3)}$. These phenomena include: 1) the inverse Faraday effect [14]; 2) the optical Faraday effect [15]; 3) light shifts in atomic spectra induced by a circularly polarized laser at visible frequencies [16]; 4) magnetization at second order in $B^{(0)}$ of an electron plasma [17] with high intensity microwave pulses.

In field-particle physics, the general theory of gauge geometries is well developed [12], and there is a need only to adapt it for the emergence of $\boldsymbol{B}^{(3)}$ in vacuum electrodynamics. The theory is developed [12] in terms of isospin indices in an abstract isospin space whose symmetry, however, is *O(3)*. By applying this theory to the physical space (1), (2) and (3) of circular indices in which Eqs. (2.13.1) are written, the *O(3)* electromagnetic field tensors emerge [6],

$$\left(G^{(1)*}\right)_{\mu\nu} = \left(F^{(1)*}\right)_{\mu\nu} - i\frac{e}{\hbar}\left(A^{(2)} \times A^{(3)}\right)_{\mu\nu},$$

$$\left(G^{(2)*}\right)_{\mu\nu} = \left(F^{(2)*}\right)_{\mu\nu} - i\frac{e}{\hbar}\left(A^{(3)} \times A^{(1)}\right)_{\mu\nu}, \tag{2.13.2}$$

$$\left(G^{(3)*}\right)_{\mu\nu} = \left(F^{(3)*}\right)_{\mu\nu} - -i\frac{e}{\hbar}\left(A^{(1)} \times A^{(2)}\right)_{\mu\nu}.$$

These generalize the usual $F_{\mu\nu}$ tensor [12] to include cross products of vector potentials. The cross product $A^{(1)} \times A^{(2)}$, for example, is not considered in the usual definition of $F_{\mu\nu}$ in the *O(2) (=U(1))* gauge group for electromagnetism, but is nevertheless *non-zero*, even in that gauge group, because [6]

$$B^{(3)*} = -i\frac{\kappa}{A^{(0)}}A^{(1)} \times A^{(2)}. \tag{2.13.3}$$

This reveals a fundamental inconsistency in the *O(2)* gauge symmetry. In the *O(3)* gauge group, on the other hand, we obtain, self-consistently from Eq. (2.13.2),

$$B^{(3)*} = -i\frac{e}{\hbar}A^{(1)} \times A^{(2)}. \tag{2.13.4}$$

Comparison of Eqs. (2.13.3) and (2.13.4) gives the charge quantization condition

$$eA^{(0)} = \hbar\kappa, \tag{2.13.5}$$

whose consistency within field theory is shown in the next section.

13.3 The Dirac Equation of One Electron in the Field

It is well known that the electron has intrinsic spin (S), which has no classical meaning. This is a result of the Dirac equation recounted on numberless occasions. It has been shown recently, however, that the interaction Hamiltonian formed between S and the electromagnetic field is [6]

$$H_{spin} = S \cdot B^{(3)} = \frac{e\hbar\sigma}{2m_0} \cdot B^{(3)}, \tag{2.13.6}$$

where $e/(2m_0)$ is the gyromagnetic ratio and σ is a Pauli spinor and is governed exclusively by $B^{(3)}$, and by no other field component. Therefore $B^{(3)}$ is to vacuum electromagnetism as S is to the electron, an intrinsic component which is not only non-zero, but irremovable. In other words, without $B^{(3)}$, the ineluctably and characteristically quantum mechanical part of the Dirac equation of the electron in the field would be entirely and incorrectly missing. The Dirac Hamiltonian eigenvalue would become identical with the classical Hamiltonian of the electron in the field.

Thus, if S be accepted, so must $B^{(3)}$.

The specific expression for $B^{(3)}$ from the Dirac equation can be written as a vector cross product [6,12],

$$B^{(3)*} = -\frac{i}{\hbar}p^{(1)} \times A^{(2)}, \tag{2.13.7}$$

or as a commutator of a transverse momentum *operator* $\hat{p}^{(1)}$ with the field vector potential $A^{(2)}$. The magnetic flux density appearing in the spin part of the Hamiltonian, H_{spin}, is independent of time, and is therefore $B^{(3)}$, because the plane waves $B^{(1)} = B^{(2)*}$ are time dependent and vanish on averaging at order one in $B^{(0)}$. The electron's intrinsic spin must interact directly with $B^{(3)}$ of the field. This is a fundamental result from the first principles of relativistic quantum field theory, and cannot be discounted as a modeling procedure. Thus $\boldsymbol{B}^{(3)}$ is the fundamental magnetizing field of electromagnetic radiation at all frequencies. It is an experimental observable, whose presence in vacuo can be detected through the $I_0^{1/2}$ dependence mentioned in the introduction.

Equation (2.13.7), defining $\boldsymbol{B}^{(3)}$ from the Dirac equation, can be obtained from Eq. (2.13.4) defining $\boldsymbol{B}^{(3)}$ independently from considerations of *O(3)* gauge geometry, through the charge quantization condition (2.13.5) in the form $\boldsymbol{p}^{(1)} = e\boldsymbol{A}^{(1)}$. The theory is therefore consistent.

13.4 Discussion

Since H_{spin} in Eq. (2.13.6) is a Hamiltonian, it is time independent, showing that $\boldsymbol{B}^{(3)}$ is a phase free, time-independent, and observable component of vacuum electrodynamics. Equation (2.13.1) relates it to the plane waves $\boldsymbol{B}^{(1)} = \boldsymbol{B}^{(2)*}$, which are complex conjugates in the basis (1), (2), (3). It follows from the well known minimal prescription,

$$p_\mu \rightarrow p_\mu + eA_\mu \, , \tag{2.13.8}$$

(the basis [18] of the Aharonov-Bohm effect) that the transverse momenta of the electron in equilibrium with the field can be represented in the same basis by the complex conjugate pairs,

$$\boldsymbol{p}^{(1)} = \boldsymbol{p}^{(2)*} \, . \tag{2.13.9}$$

In so doing, it is understood that measurable quantities are real, physical observables, as in electrodynamics in general. The electron transverse momentum is driven by the field transverse momentum in field-electron equilibrium. This requires

$$e\boldsymbol{A}^{(1)} = \boldsymbol{p}^{(1)} = \hbar\boldsymbol{\kappa}^{(1)} = i\hbar\nabla^{(1)} \, , \tag{2.13.10}$$

and taking magnitudes on both sides leads to the charge quantization condition. The electron property (orbital angular momentum) is created from the electromagnetic field, and the charge quantization condition the electron property and the field property are indistinguishable.

Therefore, although the photon is conventionally considered to be uncharged, its quantized momentum $\hbar\kappa$ is now understood to have the classical value $eA^{(0)}$, the product of two $\hat{C}$ negative quantities. At a fundamental level, therefore, the charge on the electron e becomes the *O(3)* gauge coupling parameter, the constant of proportionality between momentum and the vector potential. This is a result of the *O(3)* symmetry itself [6,12], and so in this view, the vector potential is physically meaningful. This is confirmed in the Aharonov-Bohm effect [18] which has deeply meaningful consequences, for example in vacuum topology [12]. These inferences all rest on the emergence of $\boldsymbol{B}^{(3)}$, and illustrate its central importance in field-particle theory.

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Paper 14

The Evans-Vigier Field, $\boldsymbol{B}^{(3)}$, in Dirac's Original Electron Theory: a New Theorem of Field-Fermion Interaction

Dirac's original electron theory is used to show that a classical electromagnetic field interacts with quantized fermion half integral spin through the Evans-Vigier field, $\boldsymbol{B}^{(3)} = -i(e/\hbar)\boldsymbol{A} \times \boldsymbol{A}^*$, where $\boldsymbol{A} \times \boldsymbol{A}^*$ is the conjugate product of field vector potential, $\boldsymbol{A}$, with its own complex conjugate $\boldsymbol{A}^*$; and where $e/\hbar$ is the ratio of elementary charge to Dirac constant. Dirac's theory of the electron is recovered when $\boldsymbol{A}^*$ is replaced by $\boldsymbol{A}$. However, since $\boldsymbol{A}$ is complex from d'Alembert's equation in vacuo, $\boldsymbol{B}^{(3)}$ is always non-zero. It becomes very large at low frequencies for moderate field intensity, and has several important practical applications.

14.1 Introduction

The original description by Dirac [1] of his famous theory of the electron is used in this communication to show that the classical electromagnetic field interacts with quantized fermion spin through the Evans-Vigier field [2—10],