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Paper 12

The Evans-Vigier Field $\boldsymbol{B}^{(3)}$ Interpreted as a De Broglie Pilot Field

A straightforward consideration of the antisymmetric part of the tensor of free space light intensity leads to the result $\boldsymbol{B}^{(3)}/B^{(0)} = \boldsymbol{J}^{(3)}/\hbar$, where $\boldsymbol{B}^{(3)}$ is the Evans-Vigier field [1—10], a phase free magnetic flux density of amplitude $B^{(0)}$ carried in free space by the electromagnetic wave component, and where $\boldsymbol{J}^{(3)} = \hbar\boldsymbol{e}^{(3)}$, with $\boldsymbol{e}^{(3)}$ being a unit axial vector in the propagation axis. The field $\boldsymbol{B}^{(3)}$ therefore pilots the photon angular momentum, $\boldsymbol{J}^{(3)}$. The consequences are discussed of the wave-particle duality inherent in this result, using diffraction patterns due to $\boldsymbol{B}^{(3)}$ in a double slit interferometer.

Key words: $\boldsymbol{B}^{(3)}$ Field, de Broglie pilot field.

12.1 Introduction

It has been inferred recently [1—10] that the conventional view of free space electromagnetism is incomplete, because the classical wave interpretation produces a novel phase free magnetic flux density in the vacuum, the Evans-Vigier field $\boldsymbol{B}^{(3)}$. The latter exists in free space because there exists the *electromagnetic torque* *density* $iB^{(0)}\boldsymbol{B}^{(3)*}/\mu_0 = \boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}/\mu_0$. Here μ_0 is the vacuum permeability in *S.I.* units [11] and $B^{(0)}$ the scalar amplitude of the magnetic flux density produced in free space by Maxwell's equations. In this notation, the usual energy density in free space is the dot product of complex plane waves [12],

$$U = \frac{1}{\mu_0}\boldsymbol{B}^{(1)} \cdot \boldsymbol{B}^{(2)}, \tag{2.12.1}$$

in $J\ m^{-3}$, and we work in a complex representation [13] of three dimensional space, a representation defined by the cyclically symmetric unit vector algebra,

$$\boldsymbol{e}^{(1)} \times \boldsymbol{e}^{(2)} = i\boldsymbol{e}^{(3)*}. \tag{2.12.2}$$

Since scalar light intensity, I_0 ($W\ m^{-2}$), is, in *S.I.* units [14],

$$I_0 = cU, \tag{2.12.3}$$

the *imaginary* axial vector quantity,

$$\boldsymbol{I}_A = \frac{c}{\mu_0}\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} = i\frac{c}{\mu_0}B^{(0)}\boldsymbol{B}^{(3)*}, \tag{2.12.4}$$

is the antisymmetric part of the free space light intensity tensor [5]. So the imaginary $\boldsymbol{I}_A$ is directly proportional to the *real* and physical Evans-Vigier field $\boldsymbol{B}^{(3)}$ in vacuo [6].

In this Letter, it is shown that the equation,

$$\frac{\boldsymbol{B}^{(3)}}{B^{(0)}} = \frac{\boldsymbol{J}^{(3)}}{\hbar} = \boldsymbol{e}^{(3)}, \tag{2.12.5}$$

is a straightforward consequence of the quantization of the electromagnetic field. In Eq. (2.12.5), $\hbar$ is the Dirac constant, and the real and physical

$$\boldsymbol{J}^{(3)} = \hbar\boldsymbol{e}^{(3)}, \tag{2.12.6}$$

is an angular momentum with magnitude $\hbar$ of the field as particle. In Sec. 12.2, the result (2.12.5) is derived straightforwardly from fundamentals. In Sec. 12.3, diffraction patterns are discussed qualitatively, patterns due to $\boldsymbol{B}^{(3)}$ of a circularly polarized wave passing through a double slit interferometer [15]. Finally, a discussion is pursued of the field-particle duality inherent in Eq. (2.12.5), in that $\boldsymbol{B}^{(3)}$ and $\boldsymbol{J}^{(3)}$ are directly proportional. Since $\boldsymbol{B}^{(3)}$ is produced from a cross product of vector plane wave functions $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$, it satisfies the criteria originally proposed [16] by de Broglie for pilot waves. Since $\boldsymbol{B}^{(3)}$ is phase free and is a magnetic flux density, it is referred to henceforth as the *pilot field* for $\boldsymbol{J}^{(3)} = \hbar\boldsymbol{e}^{(3)}$, and the usual term *wave-particle* is replaced by *field-particle*.

12.2 Derivation of Equation (2.12.5) from Fundamentals

Torque has the same units as energy and is the time derivative of angular momentum. Therefore, there exist in vacuum electromagnetism *torque densities* (i.e., torques per unit volume),

$$\boldsymbol{T}_V^{(3)*} := -\frac{1}{\mu_0}\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} = -i\frac{B^{(0)}}{\mu_0}\boldsymbol{B}^{(3)*}, \text{ et cyclicum,} \tag{2.12.7}$$

in which

$$\boldsymbol{m}^{(1)} = \frac{\boldsymbol{B}^{(1)}}{\mu_0}, \tag{2.12.8}$$

and so on are oscillating magnetic dipole moments of the radiation itself. Thus,

$$\boldsymbol{T}_V^{(3)*} = -\boldsymbol{m}^{(1)} \times \boldsymbol{B}^{(2)}, \tag{2.12.9}$$

in formal analogy with the definition of magnetically generated torque in electrostatics and electrodynamics [12]. However, the *real* (i. e. physical) part of $\boldsymbol{T}_V^{(3)*}$ is identically zero because the real, physical, angular momentum density, $\boldsymbol{J}_V^{(3)}$, of the beam in vacuo is constant. Thus,

$$Re\left(\boldsymbol{T}_V^{(3)*}\right) = \frac{\partial}{\partial t}\boldsymbol{J}_V^{(3)} = \boldsymbol{0}. \tag{2.12.10}$$

Now use in Eq. (2.12.10) one of the standard axioms of quantum mechanics, one based on the de Broglie relation, the axiom

$$\frac{\partial}{\partial t} = -i\frac{En}{\hbar}, \tag{2.12.11}$$

where En is energy. Since $\boldsymbol{J}_V^{(3)}$ is real, Eqs. (2.12.7), (2.12.10) and (2.12.1) give an imaginary

$$\boldsymbol{T}_V^{(3)*} = -i\frac{En}{\hbar}\boldsymbol{J}_V^{(3)*} = -i\frac{En}{\hbar V}\boldsymbol{J}^{(3)*}, \tag{2.12.12}$$

where V is the volume used to define $\boldsymbol{J}_V^{(3)*}$, and where the real $\boldsymbol{J}^{(3)*}$ now has the units of angular momentum itself rather than angular momentum density. In vacuum electromagnetic radiation, the energy density En/V is given by Eq. (2.12.1),

$$U = \frac{En}{V} = \frac{B^{(0)2}}{\mu_0}, \tag{2.12.13}$$

and so

$$\boldsymbol{T}_V^{(3)*} = -i\frac{B^{(0)}}{\mu_0}\boldsymbol{B}^{(3)*} = -i\frac{B^{(0)2}}{\mu_0\hbar}\boldsymbol{J}^{(3)*}, \tag{2.12.14}$$

from which

$$\boldsymbol{B}^{(3)} = B^{(0)}\frac{\boldsymbol{J}^{(3)}}{\hbar}, \tag{2.12.15}$$

which is Eq. (2.12.5), with $\boldsymbol{J}^{(3)} = \hbar\boldsymbol{e}^{(3)}$. The result (2.12.15), or (2.12.5), was first derived in Ref. 1, using another method, and re-derived independently in Ref. 6.

Equation (2.12.5) can be derived in another way using a straightforward adaptation of the standard expression for $\hbar\omega$ in quantum field theory [5], the Planck-Einstein light quantum hypothesis,

$$\hbar\omega = \int U dV. \tag{2.12.16}$$

Instead of the usual $U = \boldsymbol{B}^{(1)} \cdot \boldsymbol{B}^{(2)}/\mu_0$,, we use

$$U = \frac{|\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}|}{\mu_0} = \frac{B^{(0)2}}{\mu_0}, \tag{2.12.17}$$

and obtain

$$\hbar = \frac{1}{\mu_0\omega}\int |\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}|\, dV. \tag{2.12.18}$$

In the basis (2), Eq. (2.12.18) becomes

$$iJ^{(3)*} := i\hbar e^{(3)*} = \frac{iB^{(0)}}{\mu_0\omega}\int B^{(3)*}\,dV, \tag{2.12.19}$$

and rearranging,

$$B^{(3)} = \frac{\mu_0\omega}{B^{(0)}V}J^{(3)} = B^{(0)}\frac{J^{(3)}}{\hbar}, \tag{2.12.20}$$

which is again Eq. (2.12.5).

It has been shown that Eq. (2.12.5) is the direct, self-consistent result of two fundamental axioms of quantum mechanics, Eqs. (2.12.11) and (2.12.16).

12.3 Diffraction Patterns Due to $B^{(3)}$

In order to understand quantitatively the implications of Eq. (2.12.5) in field-particle duality, it is necessary to consider a Young experiment for $B^{(3)}$ carried out with a *circularly polarized* incident beam, which is diffracted through the double aperture of the interferometer to form a diffraction pattern. In classical electromagnetism, this requires an exact solution of the Maxwell equations as described recently by Jeffers *et al.* [17] for linearly polarized incident radiation. For linearly polarized radiation, however, $B^{(3)}$ nets to zero, because it changes sign from right to left circular polarization [6]. It would therefore be interesting to repeat the work of Jeffers *et al.* [17] for circularly polarized incident radiation and to map the $B^{(3)}$ diffraction patterns quantitatively and accurately. In the absence of such data we draw a qualitative sketch of the patterns to be expected using the relation between the magnitude of $B^{(3)}$ and beam intensity (I_0, in W m^{-2}),

$$B^{(3)} = B^{(0)}e^{(3)} = \left(\frac{I_0}{\epsilon_0 c^3}\right)^{1/2} e^{(3)}, \tag{2.12.21}$$

where ϵ_0 is the *S.I.* vacuum permittivity. Equation (2.12.21) expresses $B^{(3)}$ in terms of $I_0^{1/2}$, and magnetization [11,18] due to $B^{(3)}$ therefore has an $I_0^{1/2}$ profile which is observable in principle using microwave pulses to magnetize an electron plasma [18]. Fig. (9) of Jeffers *et al.* [17] shows lines of constant I_0 forming a diffraction pattern indistinguishable from an *interferogram* one which shows considerable structure [17] within a few wavelengths of the slits. This structure is unobtainable [17] in the usual scalar theory of diffraction [12], and the energy flow is calculated along paths normally interpreted as an interference pattern. However, as pointed out by Jeffers *et al.* [17], there is no such thing present as a classical interference, i.e., no radiation actually crosses the axis of symmetry. In this view, no radiation passing through the top aperture arrives at a point below the axis of symmetry and vice-versa.

Qualitatively, we expect similar patterns for the diffracted $B^{(3)}$ to be determined by Eq. (2.12.21) through the *square root* of the intensity. The significance of Eq. (2.12.21), and of the expected $B^{(3)}$ diffraction pattern, is discussed as follows.

12.4 Discussion

Although this exact, classical analysis [17] of diffraction appears to be in need of extension to incident circular polarization, in which $B^{(3)}$ is non-zero, we discuss here the inference that $B^{(3)}$ is the pilot field of $\hbar$, the photon's angular momentum. If so, $B^{(3)}$ and $\hbar$ are simultaneously measurable in the de Broglie-Vigier-Bohm interpretation [19] of the quantum theory. Lines of constant $B^{(3)}$ in a diffraction pattern would be lines of constant $\hbar e^{(3)}$ in the basis (2). These ideas do not occur in conventional electrodynamics [12] in which $B^{(3)}$ is undeveloped. The existence of $\boldsymbol{B}^{(3)}$ in vacuo [6], however, has by now been demonstrated in many ways, two of which are given for the first time in this Letter. The magnetizing effect of $\boldsymbol{B}^{(3)}$ can be demonstrated [6] using the classical Hamilton-Jacobi equation of one electron (e) in the classical electromagnetic field represented by the four-potential (A_μ), a demonstration which shows that the trajectory of the electron in the beam is governed *entirely* by $\boldsymbol{B}^{(3)}$ and by no other vacuum field. In *retrospect* it has become clear that this is due to the fact that $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}/\mu_0$ is a torque density of radiation in the vacuum, and to the fact that $\boldsymbol{B}^{(3)}$ is directly proportional to the radiation's angular momentum density (Eq. (2.12.5)). Prior to this however, $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$ was an almost unknown quantity labelled by some nonlinear opticians as the *conjugate product*, although it is only one out of several possible conjugate products of the vacuum electromagnetic field [5—11]. The obscurity of this language precluded its clear interpretation, but the magnetic conjugate product is the radiation torque density multiplied by the vacuum permeability, a torque density which is simply $iB^{(0)}\boldsymbol{B}^{(3)*}/\mu_0$. The torque per unit volume of radiation is therefore directly proportional to the real and physical Evans-Vigier field $\boldsymbol{B}^{(3)*}\left(=\boldsymbol{B}^{(3)}\right)$ of electromagnetism in the vacuum.

The use of the classical Hamilton-Jacobi equation of e in A_μ to demonstrate the existence of $\boldsymbol{B}^{(3)}$ from the principle of least action [6] is significant in at least two ways. Firstly, in a historical context, Cushing [20] has pointed out that de Broglie originally saw the classical Hamilton-Jacobi equation as providing "...an embryonic theory of the union of waves and particles, all in a manner consistent with a realist conception of matter". Equation (2.12.5) now shows that if $\hbar$ is the angular momentum of a particle, the photon, then $\hbar$ must be directly linked with $\boldsymbol{B}^{(3)}$, and in the realist view, be simultaneously observable with it. Rewriting Eq. (2.12.5),

$$\boldsymbol{J}^{(3)} = \hbar\left(\frac{\boldsymbol{B}^{(3)}}{B^{(0)}}\right), \tag{2.12.22}$$

we obtain an expression which is directly analogous with the Planck-Einstein and de Broglie relations, $En = \hbar\omega$ and $\boldsymbol{p} = \hbar\boldsymbol{\kappa}$ respectively.

Secondly, as shown by Bohm [21], the Schrödinger equation can be interpreted in the realist manner [22] by developing it into a quantized Hamilton-Jacobi equation, provided that the quantum potential is introduced, and provided that non-locality and concepts such as superluminal action at a distance are accepted as valid hypothesis. (The causal, realist point of view of Selleri *et al.* [23] does not accept action at a distance.) These questions are addressed in the interesting volume [7] recording the de Broglie centennial.

In the Copenhagen agreement [7] on the other hand, the quantum equivalent of $\boldsymbol{B}^{(3)}$ is interpreted as an angular momentum operator, $\hat{B}^{(3)}$, the photomagneton [1,4,6]. The latter is directly proportional to $\hat{J}^{(3)}$ in the vacuum, and $\hat{J}^{(3)}$ is an angular momentum operator of quantum mechanics in the Copenhagen view, and obeys the commutator relations of such operators. Is it possible to use the Evans-Vigier field to distinguish between the Copenhagen and realist interpretations of quantum mechanics? In order to begin to scratch the surface of this question, we can adapt Bohm's original discussion [2] as far as possible, in the context of diffraction patterns for $\boldsymbol{B}^{(3)}$. The line of argument is that if $\boldsymbol{B}^{(3)}$ is the de Broglie pilot wave of $\hbar$, it is simultaneously measurable with $\hbar$ in the realist view.

In the classical theory of electrodynamics [12], $\boldsymbol{B}^{(3)}$ is expected to be modified by diffraction as described accurately for the first time by Jeffers *et al.* [17]. This is a purely classical phenomenon which can be inferred by solving Maxwell's equations with the appropriate boundary conditions. The diffraction patterns after passing through the double apertures are those of $\boldsymbol{B}^{(3)}$ itself, so must be those of the angular momentum of the radiation. The latter can be represented after quantization by $\hbar\boldsymbol{e}^{(3)}$, whose magnitude is $\hbar$. The particle (photon) concomitant with the diffracted wave therefore has angular momentum magnitude $\hbar$. If so, however, where is the particle after diffraction [7]? If the incoming wave-particle is equivalent to one photon, what happens to the photon on diffraction? This question is answered entirely differently in the realist and Copenhagen views of quantum mechanics [7]. In the simpler case of light passing through a beam divider, the duality of $\hbar$ and $\boldsymbol{B}^{(3)}$ is described as follows.

In the realist interpretation [7] the photon carries particulate information, and at random goes to one of two detectors, A or B, after the light has been split by the beam divider. However, the spin field $\boldsymbol{B}^{(3)}$ goes to both detectors simultaneously, detectors which measure split beam intensity. Since single photon (and neutron) generators are now available [7], these assumptions are experimentally explorable, and Aspect *et al.* [24] appear to have shown that if a photon goes one way, there is no photon present in the other arm, there is 100% anticorrelation. Therefore the photon angular momentum, $\hbar$, if detected by A, cannot be detected simultaneously by B. (We can imagine A and B to be ultra-sensitive absorption spectrometers that can detect the absorption of $\hbar$ through atomic or molecular selection rules [14] on angular momentum.) Therefore, if $\boldsymbol{B}^{(3)}$ is simultaneously measurable with $\hbar$, then the presence of $\boldsymbol{B}^{(3)}$ at B should be simultaneously measurable with $\hbar$ arriving at A. Since $\boldsymbol{B}^{(3)}$ is proportional to the square root of intensity, then this experiment should be feasible and would show that $\boldsymbol{B}^{(3)}$ is the de Broglie pilot field [7] for $\hbar$. In this view, the wave function is the classical, Maxwellian, wave itself, and can exist simultaneously at A and B when there is one photon at A. In this interpretation, however, it is necessary to assert that the relation between electromagnetic energy density and intensity, the classical Eq. (2.12.3), holds if the empty wave containing $\boldsymbol{B}^{(3)}$ is to be observable as intensity, i.e., power per unit area. This has to be true in the *absence* of $\hbar$.

In the Copenhagen interpretation, as described by Croca [25], the light incident on the beam splitter is divided into two wave packets, and when one of these hits a detector, A, for example, the photon has chosen that particular path. The wave function is a wave of probability, and the detector A is a measuring device which has the effect of bringing the photon into observational reality. Thus, *causality is lost* and the wave of probability at B vanishes and is lost to the physical, or measurable, world. Reality in the Copenhagen view is something that *follows* measurement, and this is counter intuitive. (However intuition is by its very nature, subjective.) Therefore the photon if detected at A has been brought into measurable existence at A, and is not measurable at B simultaneously. Therefore, the wave function of the photon, when detected at A, brings it into existence at A, but before that, the photon *exists* only as a probability. In standard quantum optics, if a photon exists at A, $\boldsymbol{B}^{(3)}$ exists at A in terms of photon creation and annihilation operators [3—6]. If there is no photon at B, (i.e., if no photon is *created* at B by the measuring device) there is no $\boldsymbol{B}^{(3)}$ at B, and so nothing at all should be detectible at B, while at A we detect $\boldsymbol{B}^{(3)}$ through the square root of intensity and $\hbar$ through our ultra-sensitive spectrometric device.

This experiment appears to be a clear way of distinguishing between these two interpretations. Evidently, if $\boldsymbol{B}^{(3)}$ is an *empty field* at B, as the realists assert [7], then it must carry *classical* intensity, even though it is supposed not to carry *quantized* energy. This point of view can be sustained logically only if the intensity of an empty wave is not a function of $\hbar\omega$, the quantum of light energy known as the photon. The reason is that there is no photon at B, while there is still intensity at B in the realist point of view.

Finally, we discuss briefly the idea of $\boldsymbol{B}^{(3)}$ as a pilot field. As discussed by Bohm [21] and Vigier [26], there is an entity, ψ, guiding the particle in the wave-particle duality of de Broglie, an entity which is written in terms of the real mechanical action S as

$$\psi = R \exp\left(i\frac{S}{\hbar}\right), \tag{2.12.23}$$

so that R^2 is the *probability* that a particle of mass m have a velocity $\boldsymbol{v} = \nabla S/m$. In his paper of 1952 [21], Bohm showed that this is a plausible idea if taken to its logical conclusion, and met the objections of Pauli to de Broglie's initial proposal, published in 1930 [27]. A slight extension of the pilot wave idea is to write

$$\psi^{(1)} = \boldsymbol{R}^{(1)} e^{(iS)/\hbar} = \psi^{(2)*}, \tag{2.12.24}$$

where S is the electromagnetic action [6,21],

$$S = \hbar(\omega t - \boldsymbol{\kappa} \cdot \boldsymbol{r}), \tag{2.12.25}$$

and where $\boldsymbol{R}^{(1)}$ and $\boldsymbol{R}^{(2)}$ are in the complex circular basis (2),

$$\boldsymbol{R}^{(1)} = \boldsymbol{e}^{(1)}, \quad \boldsymbol{R}^{(2)} = \boldsymbol{e}^{(2)}. \tag{2.12.26}$$

In this picture,

$$\psi^{(0)2} = |\psi^{(1)} \times \psi^{(2)}|, \tag{2.12.27}$$

is the probability of finding a particle with an angular momentum given by

$$|\boldsymbol{J}^{(3)}| = \frac{\partial S}{\partial \phi}, \tag{2.12.28}$$

where [21], ϕ is the azimuthal angle. Therefore $\boldsymbol{B}^{(3)}$, which is directly proportional [1,6] to $\boldsymbol{J}^{(3)}$, is a pilot field of the particulate angular momentum, $\hbar$, of the photon.

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Paper 13

The Charge Quantization Condition: Link Between the O(3) Gauge Group and the Dirac Equation

The charge quantization condition (*CQC*) equates the quantized vacuum photon momentum to the classical product $eA^{(0)}$, where e is the charge on the electron and where $A^{(0)}$ is the scalar magnitude of the potential four-vector of electromagnetic radiation. It is shown that the *CQC* emerges consistently from the expression for the Evans-Vigier field $\boldsymbol{B}^{(3)}$ in the *O(3)* gauge group of vacuum electromagnetism and the Dirac equation for the spinning trajectory of an electron in the field.

Key words: charge quantization condition, $\boldsymbol{B}^{(3)}$ field

13.1 Introduction

The magnetic components of the ordinary plane waves of vacuum electromagnetism are now known [1-10] to act as the source of the magnetizing field $\boldsymbol{B}^{(3)}$, the Evans-Vigier field [6]. The real and physical $\boldsymbol{B}^{(3)}$ field propagates through the vacuum with the plane waves, and is an axial vector directed in the propagation axis. It is an experimental observable, and can be isolated [6,9] from the concomitant plane waves through its magnetization of material matter, in the simplest instance one