457

Foundations of Physics Letters, Vol. 10, No. 1, 1997

THE PHASE-FREE, LONGITUDINAL MAGNETIC COMPONENT OF VACUUM ELECTROMAGNETISM

A.E. Chubykalo, M.W. Evans[a] and R. Smirnov-Rueda[b]

Escuela de Física, Universidad Autónoma de Zacatecas
Apartado Postal C-580, Zacatecas 98068, ZAC, México

[a] *Department of Physics and Astronomy, York University*
North York, Toronto, Canada

[b]*Instituto de Ciencia de Materiales, C.S.I.C.*
Madrid, Spain

Received December 12, 1996

A charge q moving in a reference laboratory system with constant velocity $\mathbf{V}$ in the X-axis produces in the Z-axis a longitudinal, phase-free, vacuum magnetic field which is identified as the radiated $\mathbf{B}^{(3)}$ field of Evans, Vigier and others.

Key words: longitudinal waves, Evans-Vigier B(3) field, displacement current.

Several inferences have converged recently on the renewed conclusion that vacuum electromagnetism is three-dimensional, not transverse as in the received view. Longitudinal electromagnetic field components in vacuo have been inferred by Majorana [1], Dirac [2], Oppenheimer [3], and Wigner [4], who described them as phase-free. Much later, "acausal" fields of this type were given independently by Gianetto [5] and by Ahluwalia and Ernst [6]. The relativistic, three-dimensional soliton theory of Hunter and Wadlinger [7] implies the same conclusion, supported empirically. Other empirically supported theories that give longitudinal fields in vacuo include those of Recami et al. [8] and Rodrigues et al. [9]. Meszaros et al. [10] have produced a thermodynamically based theory leading to the same result, whose ramifications have also been developed by Lehnardt [11]. Dvoeglazov [12] has reviewed circa 150 papers which infer non-Maxwellian properties in vacuo. Dvoeglazov et al. [13] have discussed inconsistencies between the Joos-Weinberg and Maxwell equations. A substantial work by Chubykalo and Smirnov-Rueda [14,15] removes several well-known inconsistencies in classical electrodynamics by invoking simultaneously transverse and longitudinal components in vacuo. Munera and Guzman [16] in three recent papers, have arrived at the existence of longitudinal components and the magnetic scalar potential using a rigorous re-examination of the Lorentz condition. Finally, the theory of the $\mathbf{B}^{(3)}$ field and of the $\mathbf{B}$ cyclic equations has been presented in several recent monographs [17] which develop the subject systematically to show that in general, longitudinal solutions are linked to transverse counterparts by a new equivalence principle. In this Letter it is shown that the theory of Chubykalo and Smirnov-Rueda [14] leads directly to the $\mathbf{B}^{(3)}$ field of Evans, Vigier and others [17]. These two lines of thought converge on the same conclusion.

To see this, use Gaussian units and consider a charge q moving in a reference laboratory frame with a constant velocity $\mathbf{V}$ along the positive X-axis. Let the site of the charge at instant t be $\mathbf{r}_q$, $(x_q, 0, 0)$. Maxwell's displacement current is zero in this theory everywhere. Really, a simple charge translation in space produces alterations of field components, nevertheless, they can not be treated in terms of Maxwell's displacement current. Strictly speaking, in this case Maxwell's displacement current proportional to $\partial\mathrm{E}/\partial t$ vanishes from equation of Maxwell. This statement can be reasoned by two different ways: (*i*) $\partial\mathbf{E}/\partial t = 0$, since all field components of one uniformly moving charge are implicit time-dependent functions (time enters as a unique parameter) so that from the mathematical standpoint only total time derivative can be applied in this case whereas partial time derivative turns out to be not adequate (time and distance are not independent variables); (*ii*) a non-zero value of $\partial\mathbf{E}/\partial t$ would imply a local variation of fields in time independently of the charge position and hence would imply the expansion of those local variations through the propagation of electromagnetic waves. This would contradict the fact that one uniformly moving charge does not radiate electromagnetic field.

In this respect, it was shown in [15] that in a mathematically consistent form of Maxwell-Lorentz set of equations all partial time derivatives must be substituted by *total* ones. Only in this way all ambiguities related to the application of Maxwell's displacement current can be removed. On the other hand, it would imply a correct extension of this concept to all quasistatic phenomena. Thus, a mathematically rigorous interpretation of Maxwell's equation

$$curl\,\mathbf{H} = \frac{4\pi}{c} q\mathbf{V}\delta(\mathbf{r}-\mathbf{r}_q(t)) + \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}$$

in the case of a charge moving with a constant velocity leads to the following conclusion: In empty space outside a charge the value of *curl* **H** is equal to zero.

The law of Biot and Savart [18] gives, for this system, the magnetic field strength

$$\mathbf{H} = \frac{1}{c}\mathbf{V} \times \mathbf{E}, \tag{1}$$

where the **E** is given by [20]

$$\mathbf{E} = (1 - \beta^2)\frac{q\mathbf{R}}{R^3(1 - \beta^2 \sin^2 \theta)^{3/2}}, \tag{2}$$

where R is the distance between the charge and a point of observation (in our case, $R = \sqrt{X(t)^2 + y^2 + z^2}$, $X(t) = x - x_q(t)$).

Using Ampère's law [18] without Maxwell's displacement current gives $curl\ \mathbf{H} = \frac{4\pi}{c}\mathbf{j}$, where $\mathbf{j}$ is the conducting current density $\mathbf{j} = \varrho\mathbf{V}$. Use of Gauss's theorem [18] $div\ \mathbf{E} = 4\pi\varrho$ results in

$$curl\ \mathbf{H} = \frac{1}{c}\mathbf{V}(div\ \mathbf{E}) = \frac{1}{c}curl(\mathbf{V} \times \mathbf{E}) + \frac{1}{c}(\mathbf{V} \cdot \nabla)\mathbf{E} \tag{3}$$

(using $div\ \mathbf{V} = (\mathbf{E} \cdot \nabla)\mathbf{V} = 0$). However, from Eq. (1),

$$curl\ \mathbf{H} = \frac{1}{c}curl(\mathbf{V} \times \mathbf{E}); \tag{4}$$

and Eqs. (3) and (4) produce a paradox, because $(\mathbf{V} \cdot \nabla)\mathbf{E}$ is rigorously non-zero. There is a term needed to cancel out the first term on the right hand side of Eq. (3), which has been derived in the steady state [17] assuming that there is no change in net charge density anywhere in space, i.e., by using the Ampère's law without Maxwell's displacement current. The missing term must therefore originate in an entirely *novel* displacement current, $\mathbf{j}_d$, hitherto unconsidered in electrodynamics. Thus Ampère's law becomes

$$curl\ \mathbf{H} = \frac{4\pi}{c}(\mathbf{j} + \mathbf{j}_d). \tag{5}$$

We know that $div\, curl\ \mathbf{H} = 0$ from vector analysis [19]; so, since $\mathbf{j}_d$ is not Maxwell's famous displacement current by construction [15] (thus $div\ \mathbf{j}_d = 0$), the only possible alternative is

$$\mathbf{j}_d = \frac{1}{4\pi}curl(\mathcal{U}\mathbf{F}), \tag{6}$$

where $\mathcal{U}(x,y,z,t)$ and $\mathbf{F}(x,y,z,t)$ are scalar and vector functions of space and time. We also note that the solution (6) is part of a more general, well-known, Eq. [17]

$$div\ \mathbf{j}_d = \frac{1}{4\pi} div \left(\frac{d\mathbf{E}}{dt} \right).$$

From Eq. (3), it is seen that $\mathbf{F}$ is in the Z-axis, mutually perpendicular to V_x and E_y and has been introduced in the context of a steady state, *phase-free*, problem. Also, $\mathcal{U}\mathbf{F}/c$ has the units of magnetic field strength, which we denote $\mathbf{H}^{(3)}$. This is clearly the analogue of $\mathbf{B}^{(3)}$ [16]. Equations (3) and (4) become the same therefore if

$$curl(\mathcal{U}\mathbf{F}) = -(\mathbf{V} \cdot \nabla)\mathbf{E}. \tag{7}$$

In source-free regions of space (i.e., very far from the charge) we obtain[1]

$$curl(\mathcal{U}\mathbf{F}) = 0. \tag{8}$$

Since $\mathbf{F}$ is phase-free in vacuum, its curl is zero, and so:

$$grad\ \mathcal{U} \times \mathbf{F} = 0. \tag{9}$$

Since $\mathbf{F}$ is in the Z-axis by construction, it is given, from Eq. (9), finally, by

$$F_z = -\left(\frac{\partial \mathcal{U}}{\partial z} \right)^2 w, \tag{10}$$

---

[1] The rigorous derivation of Eq. (7) requires the separation of fields [14]:

$$\mathbf{E}_{(tot)} = \mathbf{E}_0 + \mathbf{E}^*,$$

where $\mathbf{E}_0$ becomes the solution of Poisson's equation in the static limit, and where $\mathbf{E}^*$ is the solution of the wave equation for free field. Therefore $\mathbf{E}^*$ is a function of retarded time, but $\mathbf{E}_0$ is not. This requires a careful re-examination of precepts in partial differential analysis, and we have carried this out in the course of our derivation of Eq. (7). More details was reported in [15] and will be reported in future work. Equation (7) is rigorously correct if and only if $\mathbf{E}_0$ is a function of the type $\mathcal{F}(X(T),y,z)$, where time T does not dependent on retarded time (T is not denoted by the retarded time); and if $\mathbf{E}^*$ is a function of the type $\mathcal{F}(x,y,z,t)$ where t is compound function of retarded time (t is denoted by the retarded time and vice versa).

where w is an arbitrary constant scalar.

This is a phase-free, radiated longitudinal magnetic field, which can exist in the absence or presence of Maxwell's displacement current, and which is produced by our novel displacement current $\mathbf{j}_d$.

Thus $\mathbf{F}$ has the same properties precisely as the previously inferred $\mathbf{B}^{(3)}$ magnetic flux density [16]. It is the radiated longitudinal magnetic field due to the infinitely distant charge q. Such a field does not exist in the received view in the absence of Maxwell's displacement current $\partial \mathbf{E}/\partial t$. Furthermore, since *curl* $\mathbf{F} = 0$ in vacuo, it follows that $\mathbf{F} = grad\ \varphi_m$, where φ_m is the magnetic scalar potential of Munera and Guzman [16]. Also, since *div* $\mathbf{F} = 0$ in vacuo, then $\mathbf{F} = curl\ \mathbf{A}$; and so $curl\ \mathbf{A} = grad\ \varphi_m$ in vacuo. This leads to the magnetic dual interpretation of Maxwell's equations by Munera and Guzman [16], who used the conventional displacement current. In general, $\mathbf{B}^{(3)}$ coexists with, and is linked geometrically to, the transverse irradiated wave component $\mathbf{B}^{(1)} = \mathbf{B}^{(2)*}$ [17] through the vacuum $\mathbf{B}$ cyclic equations. The transverse irradiated waves, however, are phase dependent in vacuo. The field $\mathbf{F}$ can exist when $\mathbf{E}$ (free) is not zero and $\mathbf{V} = 0$ because determinants of Eqs. (7) and (9) are zero and Eq. (9) must have a non-zero solution, even when all minors of (9) are zero. In other words, this is true even when $\mathbf{E}$ on the right hand side of Eq. (7) is zero, i.e., when the only field present is the irradiated (source-free) field. The results of our calculation are different from those of Jackson [18], p. 381, where the relativistic radiation from a charge translating with constant velocity is shown to be a plane polarized transverse wave, with an oscillating longitudinal component. Jackson uses implicitly Maxwell's displacement current because the non-zero field components resulting from his calculation are time dependent. A complete understanding of this basic problem in electrodynamics requires therefore consideration of *both* the Maxwell displacement current and our novel current $\mathbf{j}_d$. This should produce, consistently, the $\mathbf{B}$ cyclic theorem in vacuo, i.e.,

$$\mathbf{B}^{(1)} \times \mathbf{B}^{(2)} = iB^{(0)}\mathbf{B}^{(3)*} \tag{11}$$

in cyclic permutation in the basis ((1), (2), (3)) [17].

ACKNOWLEDGEMENTS

M.W.E. is grateful to the York University, Toronto, and the Indian Statistical Institute for visiting professorships. Many colleagues are thanked for e-mail discussion and preprints of related work. Authors are indebted for financial support, R. S.-R. to the Instituto de Ciencia de Materiales, C.S.I.C., Madrid, Spain and A.E.C. to the Zacatecas University, México. Authors acknowledge many stimulating discussions with Prof. V. Dvoeglazov.

REFERENCES

1. R. Mignani, E. Recami, and M. Baldo, *Lett. Nuovo Cimento* **11**, 568 (1974).
2. P.A.M. Dirac, *Directions in Physics* (Wiley, New York, 1978).
3. J.R. Oppenheimer, *Phys. Rev.* **38**, 725 (1931).
4. E.P. Wigner, *Ann. Math.* **40**, 149 (1939).
5. E. Gianetto, *Lett. Nuovo Cimento* **44**, 140 (1985).
6. D.V. Ahluwalia and D.J. Ernst, *Mod. Phys. Lett. A* **7**, 1967 (1992).
7. G. Hunter and R.L.P. Wadlinger, *Phys. Essays* **2**, 156 (1989).
8. A.O. Barut, G.D. Maccarone, and E. Recami, *Nuovo Cimento A* **71**, 509 (1982). V.S. Olkhovsky and E. Recami, *Phys. Rep.* **214**, 339 (1992). W. Heitman and G. Nimtz, *Phys. Lett. A* **196**, 154 (1994). E. Recami, *Rivista Nuovo Cimento* **9**(6) (1986).
9. W.A. Rodrigues, Jr., and J.-Y. Liu, Institute of Mathematics, State University of Campinas, Brazil, RP 12/96 (1996).
10. M. Meszaros, *Found. Phys. Lett.*, submitted for publication. M. Meszaros and P. Molnar, work in progress.
11. B. Lehnardt, in M. W. Evans, J.-P. Vigier, S. Roy, and G. Hunter, eds., *The Enigmatic Photon, New Developments*, Vol. 4 (Kluwer, Dordrecht, 1997), in preparation.
12. V.V. Dvoeglazov, *ibid.*, in preparation.
13. V.V. Dvoeglazov, Yu.N. Tyukhtyaev, and S.V. Khudyakov, *Russ. J. Phys.* **37**, 898 (1994).
14. A.E. Chubykalo and R. Smirnov-Rueda, *Phys. Rev. E* **53**(5), 5373 (1996).
15. A.E. Chubykalo and R. Smirnov-Rueda, submitted to *Phys. Rev. E*.
16. H. Munera and O. Guzman, *Found. Phys. Lett.*, in press.
17. M.W. Evans, J.-P. Vigier, S. Roy, and S. Jeffers, *The Enigmatic Photon*. Vols 1-3 (Kluwer, Dordrecht, 1994, 1995, 1996). M.W. Evans, *Physica B* **182**, 227 (1992); *Physica A* **214**, 605 (1995); *Found. Phys.* **24**, 892, 1519, 1671 (1994); 25, 175, 383 (1995); *Found. Phys. Lett.* **7**, 67, 209, 467 (1994); 8, 63, 83, 187, 363, 385 (1995); also papers in *Found. Phys. Lett.* **9** (1996).
18. J.D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1962).
19. G. Stephenson, *Mathematical Methods for Science Students*, 5th impression (Longmans & Green, London, 1968).
20. L.D. Landau and E.M. Lifshitz, *Teoria Polia* (Nauka, Moscow, 1973) [English translation: *Classical Theory of Field* (Pergamon, Oxford, 1985)].