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MANIFESTLY COVARIANT THEORY OF THE ELECTROMAGNETIC FIELD IN FREE SPACETIME, PART 3: $\hat{C}$, $\hat{P}$, AND $\hat{T}$ SYMMETRIES

I. INTRODUCTION

It has recently been observed[1–5] that there exists an equation of electrodynamics in vacuo that defines a longitudinal magnetic field, $\mathbf{B}^{(3)}$, which is independent of the phase of the electromagnetic plane wave, thus showing for the first time that there exist physically meaningful longitudinal solutions to Maxwell's equations in vacuo. Parts 1 and 2 of this series[1, 2] developed the theory of manifestly covariant electrodynamics from this basic observation, and recent work by Farahi and Evans[4] has shown that the existence of $\mathbf{B}^{(3)}$ implies the existence of its longitudinal electric counterpart $\mathrm{i}\mathbf{E}^{(3)}$. In Part 1[1] it was shown that $\mathrm{i}\mathbf{E}^{(3)}$ and $\mathbf{B}^{(3)}$ do not contribute to the electromagnetic energy density, and that Poynting's theorem can be expressed in terms of four, rather than two, polarizations. The existence of four photon polarizations, (0), (1), (2), and (3), was reconciled with two photon helicities, $+1$ and -1, by noting[1] that the helicities can be defined in terms either of (0) and (3) or of (1) and (2). Here (0) denotes the timelike photon polarization, (1) and (2) the transverse spacelike, and (3) the longitudinal spacelike. In Part (2), the Lorentz force equation was expressed in manifestly covariant form.

In this paper (Part 3), the fundamental symmetries of physics are applied to the basic equation

$$\mathbf{B}^{(3)} = \frac{\mathbf{E}^{(1)} \times \mathbf{E}^{(2)}}{\mathrm{i} E_0 c} \tag{1}$$

of manifestly covariant electrodynamics (MCE). Here $\mathbf{B}^{(3)}$ is linked[1–5] to the transverse, oscillating, electric fields $\mathbf{E}^{(1)}$ and $\mathbf{E}^{(2)}$ of the plane wave in vacuo, where c is the speed of light. Here $\mathbf{E}^{(1)}$ is the complex conjugate of $\mathbf{E}^{(2)}$:

$$\mathbf{E}^{(1)} \equiv E_0 \hat{\mathbf{e}}^{(1)} \mathrm{e}^{\mathrm{i}\phi} \tag{2a}$$

$$\mathbf{E}^{(2)} \equiv E_0 \hat{\mathbf{e}}^{(2)} \mathrm{e}^{-\mathrm{i}\phi} \tag{2b}$$

where

$$\phi = \omega t - \boldsymbol{\kappa} \cdot \mathbf{r} \tag{3}$$

is the phase of the plane wave, with, as usual, ω as the angular frequency at instant t, $\boldsymbol{\kappa}$ the wave vector at position $\mathbf{r}$. The circular basis[6, 7] is used to define the unit vectors $\hat{\mathbf{e}}^{(1)}$ and $\hat{\mathbf{e}}^{(2)}$:

$$\hat{\mathbf{e}}^{(1)} = \frac{1}{\sqrt{2}}(\mathbf{i} - \mathrm{i}\mathbf{j}) \tag{4a}$$

$$\hat{\mathbf{e}}^{(2)} = \frac{1}{\sqrt{2}}(\mathbf{i} + \mathrm{i}\mathbf{j}) \tag{4b}$$

where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in axes X and Y of the Cartesian frame (X, Y, Z).

In Section II, it is shown that Eq. (1) is invariant under the following conditions:

1. The charge conjugation operator $\hat{C}$, which changes the sign of charge in classical electrodynamics, and in particle physics produces the antiparticle from the original particle
2. The parity inversion operator $\hat{P}$
3. The motion reversal operator $\hat{T}$

In other words, the left and right sides of Eq. (1) remain balanced after application of $\hat{C}$, $\hat{P}$, and $\hat{T}$ to each variable on both sides. Equation (1) is therefore a legitimate equation of electrodynamics, and $\mathbf{B}^{(3)}$ has the $\hat{C}$, $\hat{P}$, and $\hat{T}$ symmetries, and units, of magnetic flux density. $\mathbf{B}^{(3)}$ is also a solution of Maxwell's equations[1–5] in vacuo, and is therefore a real, physically meaningful, longitudinal magnetic field with polarization (3). It has been shown in Parts 1 and 2 of this series that as a direct consequence, electrodynamics (both classical and quantum) must be made manifestly covariant in nature.

In Section II, the fundamental symmetries $\hat{C}$, $\hat{P}$, and $\hat{T}$ are applied to electromagnetic radiation in vacuo, represented by the helicity λ and the potential four vector A_μ. These are the two fundamental elements of the electromagnetic plane wave in vacuo. The helicity λ is negative to $\hat{P}$, and is a number, $+1$ or -1. In contemporary quantum field theory[8] λ is defined for the massless electromagnetic gauge field as the ratio of the Pauli-Lubansky pseudovector W_μ to the generator of spacetime translations P_μ. It is related in the lightlike condition to the second (spin) Casimir invariant of the inhomogeneous Lorentz group (or Poincaré group). The first (mass) Casimir invariant is zero for the electromagnetic field, and so λ is the only nonzero quantity that is invariant to the most general type of Lorentz transformation in the theory of special relativity. The Lorentz invariant spacetime character of the electromagnetic wave is described therefore in terms of λ. In the quantum field the photon is described by two helicities, $+1$ and -1. On the other hand, the concommitant electrodynamic properties of the electromagnetic field in vacuo are described by d'Alembert's equation:

$$\Box A_\mu = 0 \tag{5}$$

where $\Box$ is the d'Alembertian and A_μ the potential four vector. The electric and magnetic parts of the electromagnetic field can be described in terms of A_μ (Refs. 9–14). It is therefore necessary and sufficient to describe electromagnetism in vacuo in terms of the fundamental spacetime quantity λ, and the fundamental electrodynamic quantity A_μ. Section III therefore considers $\hat{C}$, $\hat{P}$, and $\hat{T}$ symmetry applied to λ and A_μ, and defines the response of the electromagnetic field to $\hat{C}$, $\hat{P}$, and $\hat{T}$ in terms of λ and A_μ. Specifically, it is shown that nonzero longitudinal solutions of Maxwell's equations are consistent with $\hat{C}$, $\hat{P}$, and $\hat{T}$ in vacuo. Finally, a detailed discussion is given of the correct way in which to apply $\hat{C}$, $\hat{P}$, and $\hat{T}$ to manifestly covariant electrodynamics, addressing some misconceptions in the recent literature.[15]

II. THE $\hat{C}$, $\hat{P}$, AND $\hat{T}$ SYMMETRIES OF THE FUNDAMENTAL EQUATION (1) OF MCE

We first note that the numerator on the right side of Eq. (1) is the antisymmetric part of the light intensity tensor[16–21] of the standard literature. It is a nonzero quantity in vacuo whose absolute magnitude is the same as the absolute magnitude of the Stokes parameter S_3 of circularly polarized light.[6] The denominator in Eq. (1) is the product of the scalar amplitude, E_0, of the electric component of the radiation with the speed of light c, and is also nonzero. The quantity $\mathbf{B}^{(3)}$ is therefore nonzero in general, provided that there is some element of circular polarity. $\mathbf{B}^{(3)}$ changes sign with the sense of circular polarity, as does $\mathbf{E}^{(1)} \times \mathbf{E}^{(2)}$ (Refs. 1–5). Since $\mathbf{B}^{(3)}$ is a magnetic field, and is physically meaningful, conventional electrodynamics becomes untenable, because it is no longer sufficient to consider just two polarizations ((1) and (2)) and to arbitrarily discard[10–14] polarization (3) as being "physically meaningless."

It is fundamentally important, therefore, to show that Eq. (1) conserves the symmetries of physics, $\hat{C}$, $\hat{P}$, and $\hat{T}$, and is therefore legitimate in all respects as an equation of electrodynamics in vacuo, because Eq. (1) must mean that conventional electrodynamics is an incomplete description, both in the classical and quantum fields.

A. $\hat{C}$ Symmetry

The charge conjugation operator $\hat{C}$ is defined as[22]

$$\hat{C}(A_\mu) = -A_\mu \tag{6}$$

and in particle physics, the photon, represented by A_μ, is negative to $\hat{C}$, being changed to the antiphoton. By definition, all spacetime quantities are unaffected by $\hat{C}$. Therefore,

$$\hat{C}(\lambda) = \lambda \tag{7}$$

From this, it follows that $\hat{C}$ changes the sign of the scalar amplitudes E_0 and B_0 of the plane wave in vacuo, and therefore changes the sign of the timelike and all spacelike components of the manifestly covariant four-vectors[1,2] E_μ and B_μ. Thus,

$$\begin{aligned} \hat{C}(\mathbf{B}^{(3)}) &= -\mathbf{B}^{(3)} \qquad \hat{C}(\mathbf{E}^{(1)} \times \mathbf{E}^{(2)}) = \mathbf{E}^{(1)} \times \mathbf{E}^{(2)} \\ \hat{C}(E_0) &= -E_0 \qquad \hat{C}(ic) = ic \end{aligned} \tag{8}$$

so that it is clear that Eq. (1) conserves $\hat{C}$ symmetry. (The $\hat{C}$ symmetry of both sides of Eq. (1) is negative.)

B. $\hat{P}$ Symmetry

The $\hat{P}$ operator,[23] parity inversion, is defined as $\hat{P}(\mathbf{r}) = -\mathbf{r}$; $\hat{P}(\mathbf{v}) = -\mathbf{v}$; where $\mathbf{r}$ and $\mathbf{v} = \dot{\mathbf{r}}$ are position and velocity, respectively. It has been shown[23] that

$$\hat{P}(\mathbf{E}^{(1)} \times \mathbf{E}^{(2)}) = \mathbf{E}^{(1)} \times \mathbf{E}^{(2)} \tag{9}$$

The $\hat{P}$ symmetry of magnetic flux density is positive,[23] and since c and $E^{(0)}$ are scalars, Eq. (1) conserves $\hat{P}$ symmetry (both sides are positive).

C. $\hat{T}$ Symmetry

The $\hat{T}$ operator,[23] motion reversal, is defined as $\hat{T}(\mathbf{r}) = \mathbf{r}$; $\hat{T}(\mathbf{v} = \dot{\mathbf{r}}) = -\mathbf{v}$; and reverses all motions in the same frame of reference. It has been shown[23] that

$$\hat{T}(\mathbf{E}^{(1)} \times \mathbf{E}^{(2)}) = -\mathbf{E}^{(1)} \times \mathbf{E}^{(2)} \tag{10}$$

The $\hat{T}$ symmetry of magnetic flux density is negative,[23] and since c and $E^{(0)}$ are scalars, they are positive to $\hat{T}$. Therefore, Eq. (1) conserves $\hat{T}$ symmetry (both sides are negative).

Therefore, Eq. (1) conserves $\hat{C}$, $\hat{P}$, and $\hat{T}$, the fundamental symmetries of physics, and is a legitimate equation of electrodynamics in vacuo. $\mathbf{B}^{(3)}$ has the $\hat{C}$, $\hat{P}$, $\hat{T}$ symmetries, and units, of magnetic flux density in vacuo, and is a solution of Maxwell's equations in vacuo. It is therefore physically meaningful, longitudinal, and phase independent.

III. THE FUNDAMENTAL SYMMETRIES OF THE ELECTROMAGNETIC PLANE WAVE

Since $\mathbf{B}^{(3)}$ (and its electric counterpart $\mathbf{E}^{(3)}$) are physically meaningful solutions of Maxwell's equations, they must be invariant to $\hat{C}$, $\hat{P}$, and $\hat{T}$, in the same way that the well-accepted, oscillating, transverse solutions (1) and (2) are invariant to $\hat{C}$, $\hat{P}$, and $\hat{T}$. The invariance of Eq. (1) is already sufficient proof that $\mathbf{B}^{(3)}$ satisfies these basic symmetry constraints in vacuo. However, a set of self-consistent rules is necessary by which the symmetries of electromagnetic radiation can be identified in terms of its most fundamental variables. We take these to be the helicity λ and the potential four vector A_μ for reasons given already.

The symmetry properties of the electromagnetic wave in vacuo can now be defined as follows:

$$\begin{aligned} [\lambda, A_\mu] &\xrightarrow{\hat{C}} [\lambda, -A_\mu] \\ [\lambda, A, \phi] &\xrightarrow{\hat{P}} [-\lambda, -A, \phi] \\ [\lambda, A, \phi] &\xrightarrow{\hat{T}} [\lambda, -A, \phi] \\ A_\mu &\equiv (A, \mathrm{i}\phi) \end{aligned} \tag{11}$$

Therefore, the $\hat{C}$ operator leaves the spacetime quantity λ unchanged by definition, while changing the sign of A_μ by definition. $\hat{C}$ thus produces a distinct entity which we identify classically as the antiwave and quantum mechanically as the antiphoton, since, by definition, $\hat{C}$ produces the antiparticle from the original particle. The antiwave is defined as the classical electromagnetic entity with the same λ as the original wave but with reversed A_μ and therefore with concommitant electric and magnetic fields of the opposite sign. The spacetime parameter λ of the antiwave is the same as that of the original wave, while the electrodynamic parameter A_μ of the antiwave is opposite in sign. This emphasizes that the antiwave is a distinct entity from the wave.

The $\hat{P}$ operator reverses the sign of λ by definition. The $\hat{P}$ symmetry of the spacelike part of A_μ (the vector potential) is negative, and that of the timelike part (the scalar potential) is positive. $\hat{P}$ again produces a distinct entity, classically the wave with opposite helicity, and quantum mechanically the photon with opposite helicity. The $\hat{T}$ operator does not change the sign of λ, and the $\hat{T}$ symmetry of the spacelike part of A_μ is negative, while that of the timelike part is positive. $\hat{T}$ again produces a distinct entity from the original wave or photon.

Therefore, distinct entities are produced by the application of all three symmetries, $\hat{C}$, $\hat{P}$, and $\hat{T}$, to the two fundamental properties, λ and A_μ of the classical electromagnetic wave or quantized photon. We denote λ and A_μ as symmetry elements of electromagnetic radiation in vacuo. Note that in the above, we have implicitly assumed that the scalar potential is nonzero, and have thus worked in a gauge such as the Lorentz gauge that allows this. In the Coulomb gauge it is assumed that the scalar potential is zero, but this loses manifest covariance unless a zero scalar potential be regarded as physically meaningful. The $\hat{C}$, $\hat{P}$, and $\hat{T}$ symmetries of a zero scalar potential are, however, the same as a nonzero scalar potential, respectively, negative, positive, and positive. In the Coulomb gauge, therefore, the $\hat{C}$, $\hat{P}$, and $\hat{T}$ symmetries are no different from a manifestly covariant gauge such as the Lorentz gauge. Conventionally, gauge invariance means that electric and magnetic fields obtained are invariant to gauge transformation. In MCE, however, it is necessary to regard the scalar potential as physically meaningful, because Eq. (1) implies the existence of four physically meaningful polarizations. Equation (1) automatically satisfies the principle of gauge invariance, because the longitudinal magnetic field $\mathbf{B}^{(3)}$ is formed from the vector product of two transverse electric fields $\mathbf{E}^{(1)}$ and $\mathbf{E}^{(2)}$ which are separately gauge invariant.

IV. DISCUSSION

In Sections II and III it has been demonstrated that Eq. (1), the key equation of manifestly covariant electrodynamics, is invariant to the fundamental symmetries of physics, and defines a quantity $\mathbf{B}^{(3)}$, which is a physically meaningful, gauge invariant, magnetic field. Equation (1) defines $\mathbf{B}^{(3)}$, a solution in vacuo of Maxwell's equations, in terms of other components of the electromagnetic plane wave in vacuo, thus showing that if the transverse components are physically meaningful, then so must be the longitudinal, making the conventional view of electrodynamics untenable, both in the classical and quantum fields.

The fundamental symmetries $\hat{C}$, $\hat{P}$, and $\hat{T}$ have been applied to the symmetry elements λ and A_μ of the electromagnetic field, and it has been shown that $\hat{C}$, $\hat{P}$, and $\hat{T}$ all produce distinct entities, or "distinct situations" by operating on the original entity or situation. The symmetry elements have been defined as λ and A_μ because these parameters are necessary and sufficient to define the spacetime and electrodynamic properties, respectively, of electromagnetic radiation in vacuo. The spacetime symmetry element λ has been chosen because it is the only nonzero Casimir invariant of the Poincaré (inhomogeneous Lorentz) group for electromagnetic radiation; and the symmetry element A_μ has been chosen because it is the only electrodynamic element that appears in d'Alembert's equation.

It is important to find a reasonable (i.e., objective) basis such as this for the definition of symmetry elements for electromagnetic radiation in vacuo. Other, arbitrary, choices of symmetry elements can (i.e., may or may not) lead to erroneous conclusions that conflict with the symmetry invariance of Eq. (1). For example, Barron[15] has recently examined the symmetry of electromagnetic radiation in vacuo using three symmetry elements, which appear to have been chosen subjectively. Since λ and A_μ are sufficient to describe the spacetime and electrodynamic properties of the radiation, Barron has one superfluous element in the three chosen, these being[15] the wave vector $\boldsymbol{\kappa}$, the sense of rotation, and the axial magnetic field $\mathbf{B}^{(3)}$. It is clear that the first two of these elements can be combined into one, the helicity, which can be regarded as a product of the linear and angular momenta of the electromagnetic radiation, and that the third, $\mathbf{B}^{(3)}$, is related to the potential four vector A_μ, and can be expressed in terms of A_μ. Barron asserts that since $\boldsymbol{\kappa}$ is unchanged by $\hat{C}$, and $\mathbf{B}^{(3)}$ is reversed in sign by $\hat{C}$, then $\mathbf{B}^{(3)}$ must be zero. In coming to this conclusion, he asserts that "the photon is its own antiphoton." However, Barron's result conflicts with our explicitly demonstrated symmetry invariance of Eq. (1), and with the fact that the numerator and denominator on the right side of Eq. (1) are both nonzero in general. His assertion that the photon is its own antiphoton conflicts with the symmetry equation

$$[\lambda, A_\mu] \xrightarrow{\hat{C}} [\lambda, -A_\mu] \tag{12}$$

i.e., $\hat{C}$ changes the sign of A_μ while leaving λ unchanged, and thus produces the antiwave (or antiphoton) from the original wave or photon. Barron's choice of three symmetry elements has therefore led to the incorrect conclusion that $\mathbf{B}^{(3)}$ is zero.

In the context of $\hat{T}$ symmetry, Barron,[15] on the other hand, concludes that $\hat{T}$ applied to his three elements does not rule out $\mathbf{B}^{(3)}$. Barron argues that $\hat{T}$ does not produce a distinct situation because the three symmetry elements he uses are all changed in sign by $\hat{T}$, and therefore $\hat{T}$ does not produce a distinct situation. However, we have seen in Section III that the use of the fundamental symmetry elements λ and A_μ produces a distinct situation when operated upon by $\hat{T}$. For this reason, $\hat{T}$ does not rule out the existence of the longitudinal field $\mathbf{B}^{(3)}$. Similarly, $\hat{P}$ acting on λ and A_μ produces a distinct situation that does not rule out $\mathbf{B}^{(3)}$. The choice of symmetry elements is therefore critically important to any argument based on $\hat{C}$, $\hat{P}$, and $\hat{T}$ symmetry that purports to show the existence or nonexistence of electric and/or magnetic fields in vacuo.

Several other arguments may be used to demonstrate why Barron's choice of symmetry elements has led to the erroneous conclusion that $\mathbf{B}^{(3)}$ is zero. These are discussed in detail as follows.

The $\hat{C}$ symmetry of all components of A_μ is negative, so that it follows by Barron's argument that oscillating transverse components such as $\mathbf{E}^{(1)}$ and $\mathbf{E}^{(2)}$ are also zero in the electromagnetic plane wave in vacuo, an erroneous conclusion. Barron's argument also implies, incorrectly, that the scalar amplitude, E_0, of the plane wave is zero for the following reason. $\hat{C}$ is a symmetry that acts on a scalar, such as charge, reversing its sign, and by definition $\hat{C}$ leaves all spacetime quantities unchanged. Any electric or magnetic field can be expressed as the product of scalar amplitude with a vector, e.g.,

$$\mathbf{E} = E_0\zeta \tag{13}$$

where ζ is a vector, a spacetime quantity. Regardless of whether ζ is transverse or longitudinal, it is unchanged by $\hat{C}$ by definition, and E_0 reverses sign by definition when operated upon by $\hat{C}$. The product

$$\hat{C}(E_0)\hat{C}(\zeta) \tag{14}$$

is therefore always negative, and it cannot be deduced on the grounds of $\hat{C}$ symmetry that in one direction the field is zero, and in orthogonal (or any other) directions nonzero, since direction, by definition, is a spacetime quantity invariant to $\hat{C}$. In manifestly covariant electrodynamics, E_0 is the timelike component of the electric field,[1,2] a nonzero quantity.

Maxwell's equations in vacuo are invariant to $\hat{C}$, $\hat{P}$, and $\hat{T}$. It follows that all legitimate solutions of Maxwell's equations in vacuo are also invariant to $\hat{C}$, $\hat{P}$, and $\hat{T}$, and Eq. (1) shows that the novel longitudinal solution $\mathbf{B}^{(3)}$ is so. The solution $\mathbf{B}^{(3)}$ cannot violate $\hat{C}$ because the equation to which it is a solution does not violate $\hat{C}$. Similarly, transverse solutions to Maxwell's equations, such as $\mathbf{E}^{(1)}$ and $\mathbf{E}^{(2)}$, conserve $\hat{C}$, $\hat{P}$, and $\hat{T}$ in vacuo. Underpinning Barron's argument is a subjective choice of three symmetry elements, described already, and the assumption that the photon and antiphoton are the same in all respects, i.e., one is not "distinct" from the other. We have argued that the photon and antiphoton are different entities, i.e.:

$$\underset{\text{Photon}}{[\lambda, A_\mu]} \xrightarrow{\hat{C}} \underset{\text{Antiphoton}}{[\lambda, -A_\mu]} \tag{15}$$

and that the choice of λ and A_μ as symmetry elements is rooted in contemporary theory of electromagnetic radiation. Only two elements are needed to define the symmetry of electromagnetic radiation in vacuo, one being an invariant of the Poincaré group, the other being the single variable of the d'Alembert equation.

Barron, therefore, bases his argument[15] on the assumption that the photon and antiphoton are indistinct, so that in an indistinct situation all variables must be relatively the same, so that $\mathbf{B}^{(3)}$ relative to $\boldsymbol{\kappa}$ must not change when both are acted upon by $\hat{C}$. We argue that $\hat{C}$ operates to produce a distinct situation, embodied in the antiwave, or antiphoton, and in a distinct situation, it is no longer reasonable to expect that all variables must be relatively unchanged, so that $\mathbf{B}^{(3)}$ may change sign with respect to $\boldsymbol{\kappa}$, and so may E_0, $\mathbf{E}^{(1)}$, and $\mathbf{E}^{(2)}$. Even within the framework of his own argument, Barron has shown only that *either* $\mathbf{B}^{(3)}$ or $\boldsymbol{\kappa}$ must be zero, so that on his grounds $\boldsymbol{\kappa}$ may be zero and $\mathbf{B}^{(3)}$ nonzero. It is therefore not possible to assert unequivocally, even within his own argument, that $\mathbf{B}^{(3)}$ is zero.

Barron proceeds to argue, on the basis of his three symmetry elements and on the basis of his assertion that the photon and the antiphoton are distinct, that $\hat{C}$ symmetry does not imply that the inverse Faraday effect,[24] magnetization by a circularly polarized laser, cannot exist. Before commenting on Barron's viewpoint in this context, we note that the inverse Faraday effect is accommodated straightforwardly by Eq. (1). This is because $\mathbf{E}^{(1)} \times \mathbf{E}^{(2)}$ interacts with a $\hat{C}$-negative rank three property tensor to induce a magnetic dipole moment.[25] Similarly, $\mathbf{B}^{(3)}$ interacts with a $\hat{C}$-positive rank two molecular property tensor (the susceptibility) to induce a magnetic dipole moment.[26]

In Barron's viewpoint, on the other hand, $\mathbf{B}^{(3)}$ is zero, so that by Eq. (1), $\mathbf{E}^{(1)} \times \mathbf{E}^{(2)}$ is zero. Despite this, his argument asserts that there is a nonzero inverse Faraday effect, showing conclusively that his viewpoint is illogical. The root error in Barron's approach is that under $\hat{C}$, both $\mathbf{E}^{(1)}$ and $\mathbf{E}^{(2)}$ are separately negative, because they are electric fields of the antiwave, which is distinct from the wave. His assertion that the wave and antiwave are in all respects identical (i.e., "indistinct") implies in his view that $\mathbf{E}^{(1)}$ and $\mathbf{E}^{(2)}$ must be separately zero, and that the product $\mathbf{E}^{(1)} \times \mathbf{E}^{(2)}$ is zero. This means that there is no inverse Faraday effect, in direct conflict with experimental data.[24] It may be argued in Barron's favor[27] that $\mathbf{E}^{(1)} \times \mathbf{E}^{(2)}$, and quantities such as $\mathbf{E}^{(1)} \times \mathbf{B}^{(2)}$, do not change sign with $\hat{C}$, but this is spurious, because neither is an electric or a magnetic field. On these albeit spurious grounds it can be similarly asserted that $\mathbf{B}^{(3)}$ is nonzero because quantities such as $\mathbf{B}^{(3)} \times \mathbf{E}^{(1)}$ and $\mathbf{B}^{(3)} \times \mathbf{B}^{(1)}$ do not change sign under $\hat{C}$, so that $\mathbf{B}^{(3)}$ is nonzero.

It is clear that energy is invariant to $\hat{C}$ and also that, in our viewpoint, the interaction energy of the antiwave with antimatter is identical with the interaction energy of the wave and matter, for example:

$$\begin{aligned} \hat{C}(\boldsymbol{\mu} \cdot \mathbf{E}^{(1)}) &= (-\boldsymbol{\mu}) \cdot (-\mathbf{E}^{(1)}) = \boldsymbol{\mu} \cdot \mathbf{E}^{(1)} \\ \hat{C}(\mathbf{m} \cdot \mathbf{B}^{(3)}) &= (-\mathbf{m}) \cdot (-\mathbf{B}^{(3)}) = \mathbf{m} \cdot \mathbf{B}^{(3)} \end{aligned} \tag{16}$$

where $\boldsymbol{\mu}$ is an electric and $\mathbf{m}$ a magnetic dipole moment. It is therefore quite natural in our argument that the inverse Faraday effect and similar effects may exist in nature, because they are governed by an interaction energy that is invariant under the basic symmetries of physics, $\hat{C}$, $\hat{P}$, and

$\hat{T}$. Thus, in the inverse Faraday effect, for example, $\mathbf{B}^{(3)}$ forms an interaction energy with an electronic magnetic dipole moment, and $\mathbf{E}^{(1)} \times \mathbf{E}^{(2)}$ with an electronic antisymmetric polarizability, both types of interaction energy being invariant under $\hat{C}$, $\hat{P}$, and $\hat{T}$.

In Barron's[15] view, however, the argument is convoluted and obscure, because if the wave be indistinct from the antiwave, as in his view, the interaction energy of the antiwave with antimatter, produced by $\hat{C}$ from matter, is no longer invariant under $\hat{C}$. Obviously, matter must be distinct from antimatter, but if wave be indistinct from antiwave, it follows that the interaction energy of antiwave with antimatter is opposite in sign to the interaction energy of wave with matter, an insupportable conclusion because all forms of energy must be indistinct under the basic symmetries of physics. Thus, if matter be distinct from antimatter, wave must be distinct from antiwave, as in our argument. Barron does not consider interaction energy in his paper,[15] but uses the fact that the induced magnetic dipole moment is $\hat{C}$ negative, which is also naturally accommodated within our argument, and also in that of Woźniak,[25] which Barron does not dispute.

There are several considerations of classical electrodynamics, for example, in the classic text by Jackson,[10] that appear to conflict with the assertion by Barron, that all forms of longitudinal solutions to Maxwell's equations in vacuo are zero. The following examples are mentioned briefly, but there are several more available.[10]

The Maxwell equations in vacuo have spherical solutions,[10] which in general are not transverse, as in plane wave solutions. The most general form of these solutions is given by Jackson's equation (16.35):

$$B = \sum_{l,m} \left[A_{lm}^{(1)} h_l^{(1)}(kr) + A_{lm}^{(2)} h_l^{(2)}(kr) \right] Y_{lm}(\theta, \phi) \tag{17}$$

where A_{lm} are arbitrary constant vectors; where $h_l^{(1)}$ and $h_l^{(2)}$ are Hankel functions, and where Y_{lm} are spherical harmonics. The longitudinal component is found for $\theta = 0$:

$$B_{\theta=0} = \sum_{l} \left[A_l^{(1)} h_l^{(1)}(kr) + A_l^{(2)} h_l^{(2)}(kr) \right] \left(\frac{2l + l}{4\pi} \right)^{1/2} \tag{18}$$

and this is clearly not zero in vacuo, being a special case of the general spherical solution (17) of the vacuum Maxwell equations:

$$\begin{aligned} (\nabla^2 + \boldsymbol{\kappa}^2)\mathbf{B} &= 0 \\ \nabla \cdot \mathbf{B} &= 0 \end{aligned} \tag{19}$$

Transverse solutions are also special cases of Eq. (17), cases that are described, for example, in Jackson's equation (16.42).[10] It is impossible to assert on the grounds of $\hat{C}$ symmetry that the fields defined in Eq. (18) are zero, while those in Eq. (16.42) of Ref. 10 are nonzero, because both Eqs. (18) and (16.42) of Ref. 10 are special cases of Eq. (17).

These are longitudinal solutions of Maxwell's equations in conducting media, where they are augmented by Ohm's law:

$$\mathbf{J} = \sigma E \tag{20}$$

where $\mathbf{J}$ is a current and σ the conductivity. The effect of $\hat{C}$ on Ohm's law is as follows:

$$-\mathbf{J} = \sigma(-E) \tag{21}$$

i.e., the conductivity is $\hat{C}$-positive. Similarly, electric permittivity and magnetic permeability in a conductor are both $\hat{C}$-positive, so changing conducting matter to conducting antimatter by operating with $\hat{C}$ does not change conductivity, permittivity, and permeability. Thus, Maxwell's equations and Ohm's law in conducting anti matter are the same as in conducting matter, and longitudinal solutions exist in both situations. The interaction of the antiwave with conducting antimatter is therefore the same as that of wave and conducting matter, as in our argument given already for the inverse Faraday effect. In Barron's view there is no antiwave, and the interaction is distinct, an insupportable conclusion.

Longitudinal, but phase dependent, solutions of Maxwell's equations exist in waveguide theory,[10] through equations such as

$$B_Z = B_0 = B_0 \cos\left(\pi \frac{x}{a} \right) e^{i\phi} \tag{22}$$

which are $\hat{C}$-invariant. Here a is a waveguide dimension and x a coordinate in this dimension. The only quantity on the right side of this equation that changes sign with $\hat{C}$ is the magnetic flux density amplitude B_0. All others are spacetime quantities which are invariant to $\hat{C}$ by definition. If we take $a \to \infty$, then for finite x,

$$B_Z \xrightarrow[a \to \infty]{} B_0 e^{i\phi} \tag{23}$$

i.e., at a point x inside a waveguide of infinite dimension a, the longitudinal magnetic field B_Z is nonzero. Since all parameters in Eq. (22) are $\hat{C}$-invariant except B_0, then according to Barron's view $\hat{C}$ operating on B_Z

of the waveguide does not produce a distinct situation, and so B_Z must vanish, an erroneous conclusion that conflicts with waveguide theory. Furthermore, if $a \to \infty$ in the waveguide, the wave B_Z is effectively propagating in a container of infinite volume, i.e., free space, and so in this situation B_Z remains nonzero in the free space limit, obtained by setting $a \to \infty$. According to Barron's view, B_Z is zero.

A closely related situation is that of resonant cavities, which again support longitudinal, phase-dependent solutions of Maxwell's equations, as in the example of a right cylindrical cavity, in which the longitudinal electric field is

$$E_Z = E_0 J_0\left(2.405\frac{\rho}{R}\right)e^{-i\omega t} \tag{24}$$

Here,[10] J_0 is a Bessel function, ρ is a point on the radius R of the cylinder, and E_0 is a scalar electric field strength amplitude. Again, the only quantity on the right side of Eq. (24) that changes sign with $\hat{C}$ is E_0, so that all properties of the cavity are invariant to $\hat{C}$. Therefore, applying $\hat{C}$ in Barron's view produces an indistinct situation in which the wave vector of the field E_Z has not changed, no cavity property has changed, but E_Z has changed. So in Barron's view E_Z is zero, an incorrect conclusion which conflicts with the theory of resonant cavities.

APPENDIX: $\hat{C}\hat{P}\hat{T}$ THEOREM

In the text of the paper, it has been shown that $\mathbf{B}^{(3)}$ does not violate any of the three discrete symmetries. $\hat{C}$, $\hat{P}$, and $\hat{T}$, and by Eq. (1), is nonzero if $\mathbf{E}^{(1)} \times \mathbf{E}^{(2)}$ is nonzero. It follows that the nonobservation of $\mathbf{B}^{(3)}$ would violate $\hat{C}\hat{P}\hat{T}$, striking at the roots of quantum theory. $\hat{C}\hat{P}\hat{T}$ theorem implies that any quantum (and by implication classical) theory of fields that is compatible with special relativity and assumes only local interactions does not violate $\hat{C}\hat{P}\hat{T}$.[8] Therefore, if a physically meaningful magnetic flux density, $\mathbf{B}^{(3)}$, is inviolate of $\hat{C}$, $\hat{P}$, *and* $\hat{T}$ separately, and is nonzero, it must be an observable by the $\hat{C}\hat{P}\hat{T}$ theorem. If it is a nonobservable, the $\hat{C}\hat{P}\hat{T}$ theorem is violated. If $\mathbf{B}^{(3)}$ is observable experimentally, on the other hand, it provides evidence for the manifest covariance of electrodynamics and conservation of $\hat{C}\hat{P}\hat{T}$.

In this context, we note that if $\mathbf{B}^{(3)}$ is an observable, and if it is inviolate of $\hat{P}$ and $\hat{T}$, and therefore of $\hat{P}\hat{T}$, then the $\hat{C}\hat{P}\hat{T}$ theorem shows that it cannot violate $\hat{C}$. Barron's argument[15] is therefore shown to be incorrect, because he has assumed that $\mathbf{B}^{(3)}$ is an observable, and has himself concluded (albeit in a subjective argument) that in consequence $\mathbf{B}^{(3)}$ does not violate $\hat{P}\hat{T}$. It follows that if $\hat{P}\hat{T}$ is conserved, and $\mathbf{B}^{(3)}$ is an observable, as assumed by Barron[15]; then it cannot violate $\hat{C}$. Thus, if $\mathbf{B}^{(3)}$ is an observable, it must conserve $\hat{C}\hat{P}\hat{T}$. Conversely, conservation of $\hat{C}\hat{P}\hat{T}$ means that $\mathbf{B}^{(3)}$ must be an observable.

It follows that the conventional electrodynamical notion that the longitudinal solutions of Maxwell's equations $\mathbf{B}^{(3)}$ and $i\mathbf{E}^{(3)}$ (Ref. 4), be "unphysical" implies $\hat{C}\hat{P}\hat{T}$ violation, thus putting in doubt the fundamentals of quantum field theory applied to the electromagnetic field. Either $\mathbf{B}^{(3)}$ is an observable and $\hat{C}\hat{P}\hat{T}$ is conserved, or $\mathbf{B}^{(3)}$ is a nonobservable and $\hat{C}\hat{P}\hat{T}$ is violated. In other words, the only possible reason why the left side of Eq. (1) is not equal to the right side is if $\mathbf{B}^{(3)}$ violated $\hat{C}\hat{P}\hat{T}$. Therefore, quantum and classical electromagnetic field theory implies that the left and right sides of Eq. (1) must be equal, and in this field theory, since $\mathbf{E}^{(1)} \times \mathbf{E}^{(2)}$ is physically meaningful, then $\mathbf{B}^{(3)}$ must be physically meaningful. Otherwise, electromagnetic field theory is fundamentally flawed.

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*THE ELECTROSTATIC AND MAGNETOSTATIC FIELDS GENERATED BY LIGHT IN FREE SPACE**

I. INTRODUCTION

The phenomenological equations of J. C. Maxwell form the basis of the classical understanding of light. The equations were formulated in the mid nineteenth century, before relativity was fully developed, and before the quantum theory came into existence. They were later put on a microscopic basis by H. A. Lorentz in his theory of the electron, and have become the starting point of a vast number of contemporary papers on the nature of light in free space and in materials. In this paper we show that there exist novel electro and magnetostatic fields in the propagation axis of the classical electromagnetic plane wave, fields that propagate in free space and conserve the structure of the well-defined Poynting vector, and therefore do not affect the law of conservation of electromagnetic energy in free space. It is usually assumed that the following are solutions to the free space Maxwell equations for a completely circularly polarized plane wave:

$$\mathbf{E}(\mathbf{r}, t) = \frac{1}{\sqrt{2}} E_0(\mathbf{i} + \mathrm{i}\mathbf{j})e^{\mathrm{i}\phi} \tag{1}$$

$$\mathbf{B}(\mathbf{r}, t) = \frac{1}{\sqrt{2}} B_0(\mathbf{j} - \mathrm{i}\mathbf{i})e^{\mathrm{i}\phi} \tag{2}$$

*R. Gauthier and F. Farahi of UNCC are thanked for suggesting the possibility of $\mathbf{E}_\Pi$.

Here E_0 is the scalar electric field strength amplitude, and B_0 the scalar magnetic flux density amplitude, **i** and **j** are unit vectors in X and Y of the laboratory frame, and ϕ is the phase of the plane wave. These solutions are oscillatory and time and space dependent through the phase

$$\phi = \omega t - \boldsymbol{\kappa} \cdot \mathbf{r} \tag{3}$$

where ω is the angular frequency of the wave, t the time, $\boldsymbol{\kappa}$ the wave vector, and **r** a position vector as usual. A whole literature is available concerning their properties.

However, the equations

$$\mathbf{E}^{\mathrm{G}} = \mathbf{E}(\mathbf{r}, t) + \mathbf{E}_\Pi \tag{4}$$

$$\mathbf{B}^{\mathrm{G}} = \mathbf{B}(\mathbf{r}, t) + \mathbf{B}_\Pi \tag{5}$$

are also valid solutions to the free space Maxwell equations. Here $\mathbf{E}_\Pi$ and $\mathbf{B}_\Pi$ are uniform, time-independent, electric and magnetic fields directed in the propagation axis Z of the plane wave. It appears always to have been implicitly assumed that $\mathbf{E}_\Pi$ and $\mathbf{B}_\Pi$ are both zero in free space, and that there is no component in Z of the plane wave in vacuo. There is no mathematical reason for this supposition, however, and as we shall see, the vectors $\mathbf{E}_\Pi$ and $\mathbf{B}_\Pi$ can be related to the well-known $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$. The source of $\mathbf{E}_\Pi$ and $\mathbf{B}_\Pi$ is therefore the same as the source of $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$. If the latter are nonzero, then so are both $\mathbf{E}_\Pi$ and $\mathbf{B}_\Pi$, in general.

Section II introduces $\mathbf{B}_\Pi$ using the imaginary conjugate product,

$$\mathbf{\Pi}^{(\mathrm{A})} \equiv E_0 c \,\mathrm{Im}(\mathbf{B}_\Pi) = \mathbf{E}(\mathbf{r}, t) \times \mathbf{E}^*(\mathbf{r}, t) = -\mathrm{i}E_0^2\mathbf{k} \tag{6}$$

of the electromagnetic plane wave,[1-8] where $\mathbf{E}^*(\mathbf{r}, t)$ is the complex conjugate of $\mathbf{E}(\mathbf{r}, t)$, i.e.,

$$\mathbf{E}^*(\mathbf{r}, t) = \frac{1}{\sqrt{2}} E_0(\mathbf{i} - \mathrm{i}\mathbf{j})e^{-\mathrm{i}\phi} \tag{7}$$

We see in Appendix A that the real and imaginary parts of $\mathbf{B}_\Pi$ are the same.

The law of conservation of energy for a plane wave in free space can be expressed through the continuity equation:

$$\boldsymbol{\nabla} \cdot \mathbf{N} = -\frac{\partial U}{\partial t} \tag{8}$$