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Relativistic theory of polarization and magnetization due to an electromagnetic field. Part 1: the velocity dependence of optical rotatory dispersion

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Abstract. A relativistic theory of polarization and magnetization due to an electromagnetic field is developed in powers of the electric and magnetic components, **E** and **B**. For an observer in the rest frame of an earthbound laboratory the observable birefringence using the light from a distant galaxy as a source of radiation becomes dependent on the velocity of that source through the Lorentz transformations. Circular birefringence and dichroism, angle of rotation, and related effects for the same optically active material using earthbound and galactic sources would provide data dependent on the velocity of the source frame with respect to the rest frame. This amplifies the birefringence to the point where tiny parity-violating effects may become observable in the rest frame of the earthbound observer.

1. Introduction

In the classical, relativistic, theory of fields [1] the Lorentz transformations provide the relations between the electric (**E**) and magnetic (**B**) components of an electromagnetic field in the rest frame and another inertial frame translating with respect to the former with a velocity v_z along its z axis. The Lorentz equations show that the electromagnetic radiation reaching the rest frame of an earthbound observer from a source in a distant galaxy contains information about the velocity v_z at which that source is receding from the Earth. In this case v_z is a significant fraction of the velocity of light c. The further away the galactic source in an expanding universe, the closer the fraction v/c approaches unity, as measured through the well known red-shift phenomenon [2].

The red shift is only one out of many optical phenomena which can be observed in the earthbound rest frame using a source of radiation in a galaxy moving away from the earth near the speed of light. Another, potentially very interesting, phenomenon is circular birefringence, which can be amplified by gathering the source radiation with an instrument such as an orbiting telescope, passing it through a plane polarizer, and then through chiral material with a rotation strength of some thousands of degrees, such as a single crystal of a helical biomacromolecule, and finally through the analyser of a polarimeter. The circular birefringence will be significantly and measurably different using the galactic source in comparison with a source in the rest frame of an earthbound laboratory. The difference contains information on v_z, and as shown in this paper, amplifies the optical activity, and thereby the tiny molecular-parity violating effects due to weak current interactions caused by handed elementary nuclear particles [3–6]. These parity-violating phenomena cause minute optical rotations in achiral heavy-metal vapours [7, 8], and the enantiomeric energy inequivalence [9].

2. The dependence of polarization and magnetization on v_z

The polarization and magnetization of an ensemble interacting with an electromagnetic field depend on its electric (**E**) and magnetic (**B**) components through an interaction Hamiltonian in the rest frame. Neglecting parity violation [3–6] this Hamiltonian is

$$\Delta H = -\boldsymbol{\mu}\cdot E - \mathbf{m}\cdot\mathbf{B}, \tag{1}$$

where $\boldsymbol{\mu}$ is the total molecular electric dipole moment and $\mathbf{m}$ its magnetic counterpart, both defined in the rest frame. The polarization is the ensemble average $\langle\boldsymbol{\mu}\rangle$ and the magnetization is $\langle\mathbf{m}\rangle$. In structurally chiral ensembles both $\boldsymbol{\mu}$ and $\mathbf{m}$ become dependent on **E** and **B** through the series expansions [10–12]

$$\mu_i = \mu_{0i} + \alpha_{1ij}E_j + \alpha^*_{2ij}B_j + \frac{1}{2!}(\beta^*_{1ijk}E_jE_k + \beta_{2ijk}E_jB_k + \beta_{3ijk}B_jE_k + \beta^*_{4ijk}B_jB_k) + \dots, \tag{2}$$

$$m_i = m_{0i} + a_{1ij}B_j + a^*_{2ij}E_j + \frac{1}{2!}(b_{1ijk}B_jB_k + b^*_{2ijk}B_jE_k + b^*_{3ijk}E_jB_k + b_{4ijk}E_jE_k) + \dots. \tag{3}$$

The multiplying coefficients are molecular-property tensors, those marked with asterisks are negative to parity-reversal and vanish in structurally-achiral ensembles [10–12] in the absence of parity P violation.

These series are valid for a source of electromagnetic radiation in the same inertial frame as the ensemble and the observer. If, however, the source translates away along the z axis of the rest frame at a velocity **v**, **E** and **B** become dependent on **v** through the Lorentz transformations of classical, relativistic field theory [1, 13]

$$\mathbf{E}' = \xi(\mathbf{E} + \mathbf{v}\times\mathbf{B}), \tag{4 a}$$

$$\mathbf{B}' = \xi[\mathbf{B} - (\mathbf{v}\times\mathbf{E})/c^2], \tag{4 b}$$

$$\xi = [1 - v^2/c^2]^{-1/2}. \tag{4 c}$$

Here the primes denote quantities in the translating frame. Thus $\mathbf{E}'$, for example, originating in the translating frame, is seen as $\xi(\mathbf{E} + \mathbf{v}\times\mathbf{B})$ in the rest frame. *The latter is the earthbound frame containing the apparatus, sample, and observer*. The **E** and **B** components of an electromagnetic field originating in the translating frame are observed in the rest frame as

$$\mathbf{E} = \xi(E_x + v_zB_y)\mathbf{i} + \xi(E_y - v_zB_x)\mathbf{j} + E_z\mathbf{k}, \tag{5}$$

$$\mathbf{B} = \xi\left(B_x - \frac{v_z}{c^2}E_y\right)\mathbf{i} + \xi\left(B_y + \frac{v_z}{c^2}E_x\right)\mathbf{j} + B_z\mathbf{k}. \tag{6}$$

3. Relativistic effects on optical phenomena in the rest frame

The polarization $\langle\boldsymbol{\mu}\rangle$, and magnetization $\langle\mathbf{m}\rangle$ are seen, by substituting equations (5) and (6) into the series (2) and (3), to tend to infinity when $v_z = c$ as a direct consequence of the Lorentz transformation. As v_z approaches c the y and x components of **E** and **B** become much larger than the respective z components as seen in the rest frame (an orbiting space telescope for example). This result is true for all electromagnetic frequencies. All optical effects described [10–12] by the series (2) and (3) will be affected likewise. The linear Rosenfeld optical rotation [14], for example, accompanies the electric polarization due to the first two terms of the series

(2), a polarization which becomes infinite when $v_z = c$ because $\mathbf{E}$ and $\mathbf{B}$ become infinite. Nonlinear effects described by the series (2) and (3) are also amplified in relativity theory. Among those in the literature are:

(1) the polarization due to $\beta^*_{4ijk}B_jB_k$, the 'magneto-chiral effect' [15–18];
(2) the magnetization due to $b_{4ijk}E_jE_k$, the 'inverse Faraday' effect' [19, 20];
(3) the magnetization due to $b^*_{2ijk}\mathrm{B}_j\mathrm{B}_k$ and $b^*_{3ijk}E_jB_k$, the 'inverse magneto-chiral effect' [21].

Other terms in the series (2) and (3) not noted in the literature are likewise amplified as v_z approaches c.

4. The amplification of *P*-violating effects

Parity P violation in molecular matter [3–6] can be caused by electron–nucleus interactions mediated by the neutral intermediate vector boson, recently observed experimentally [22]. These P-violating effects change [12] the interaction Hamiltonian (1) to

$$\Delta H = -\boldsymbol{\mu}\cdot\mathbf{E} - \mathbf{m}\cdot\mathbf{B} - \xi\boldsymbol{\mu}\cdot\mathbf{m}, \tag{7}$$

where ξ is a scalar, negative to time reversal T. The extra P-violating term is negative to P and depends on $\mathbf{E}$ and $\mathbf{B}$ of the electromagnetic field through the expansion of $\boldsymbol{\mu}$ and $\mathbf{m}$ in terms of the molecular property tensors. Thus

$$\boldsymbol{\mu}\cdot\mathbf{m} = (\mu_{0i} + \alpha_{1ij}E_j + \alpha^*_{2ij}B_j + \ldots)(m_{0i} + a_{1ij}B_j + a^*_{2ij}E_j + \ldots), \tag{8}$$

and the P-violating interaction energy depends on v_z in a relativistic context. As a direct consequence of the Lorentz transformation, the P-violating energy becomes very large in the rest frame as v_z approaches c. The P-violating term in (7) works through into the polarization and magnetization as described elsewhere [12]. For a polarization linear in $\mathbf{E}$, for example, we have

$$\mu_i = (\alpha_{1ij} + \xi(\alpha_{1ij}a_{2ij})^*)E_j, \tag{9}$$

which causes an optical rotation exclusively due to the polarization from P violation. The term theoretically becomes *infinite* when $v_z = c$ due simply to the fact that $\mathbf{E}$ becomes infinite in the rest frame as a direct consequence of the Lorentz transformation. This argument assumes only that $\mathbf{E}$ is finite in the translating frame in which the electromagnetic radiation originates.

5. Circular birefringence and dichroism in terms of v_z

In this section the Maxwell equation

$$\frac{1}{\mu_0}\nabla\times\left(\mathbf{B} - \frac{\mathbf{v}\times\mathbf{E}}{c^2}\right) = \epsilon_0\frac{\partial}{\partial t}(\mathbf{E} + \mathbf{v}\times\mathbf{B}) + N\frac{\partial}{\partial t}\left[\alpha_{1ij}(\mathbf{E} + \mathbf{v}\times\mathbf{B})_j + \alpha_{2ij}\left(\mathbf{B} - \frac{\mathbf{v}\times\mathbf{E}}{c^2}\right)_j\right], \tag{10}$$

is solved for the circular birefringence, dichroism and angle of rotation of a source of velocity v_z, assumed to be significant with respect to c. In equation (10) μ_0 is the vacuum permeability, ϵ_0 the vacuum permittivity, and N the number of molecules per metre in the chiral sample in the rest frame. Note carefully that the molecular polarizability tensor $\boldsymbol{\alpha}_1$ and optical activity tensor $\boldsymbol{\alpha}_2$ appearing in this equation are *rest-frame properties*, while the electric and magnetic fields of the galactic source are *moving-frame properties* which have been transformed into the rest frame with the Lorentz equation.

Standard methods of solution [13, 14] lead to the following equations in terms of v_z. These appear to be of direct practical use in X-ray, optical, infrared and radio telescope, which can be used to gather radiation from a galactic source receding at a velocity v_z from the Earth. Using chiral samples, the velocity v_z can be measured, for example, and the frequency dependence of the following quantities leads to velocity-dependent optical rotatory dispersion. Using an achiral sample, any observed angle of rotation for v_z close to c indicates parity violation in the sample, which is greatly amplified by the use of a source moving relative to the sample with a velocity close to c.

5.1. *Circular birefringence,* y *axis*

This is given by the ensemble averaged result

$$\langle n'_{\mathrm{R}} - n'_{\mathrm{L}} \rangle = 2\mu_0 c N \langle \alpha''_{2yy} \rangle \left(\frac{c + v_z}{c - v_z} \right), \tag{11}$$

which becomes infinite when $v_z = c$. Any parity-violating effects contained within the imaginary component $\langle \alpha''_{2yy} \rangle$ of the complex optical-rotation (Rosenfeld) tensor will therefore be amplified considerably. The result (11) reduces to the standard expression for circular birefringence [14] when $v_z = 0$.

5.2. y *axis dichroism, power absorption coefficient* α

The equivalent dichorism is, from the absorption index (the imaginary part of the refractive index),

$$\langle \alpha_{\mathrm{R}} - \alpha_{\mathrm{L}} \rangle_y = -4\mu_0 \omega N \langle \alpha'_{2yy} \rangle \left(\frac{c + v_z}{c - v_z} \right). \tag{12}$$

5.3. y *axis is angle of rotation*

The angle of rotation of radiation plane-polarized in the y axis before passing through the sample, is

$$\Theta = 2 l \mu_0 N \omega \langle \alpha''_{2yy} \rangle \left(\frac{c + v_z}{c - v_z} \right), \tag{13}$$

where l is the sample length in metres.

These expressions are all of direct practical use, either in measuring the velocity of the source v_z, or in detecting amplified P-violating effects in vapours.

Note that these quantities are, in general, different in the x axis and become equivalent only when $v_z = 0$. Relativity therefore introduces a kind of 'elliptical' polarization.

6. Discussion

The limit $v_z = c$ cannot be reached practically because radiation from the source would never reach the rest frame, simply because electromagnetic radiation propagates at c. To obtain v_z as a significant fraction of c, the source radiation must be observed in the rest frame from a far galaxy, some millions of light years distant. With an orbiting space telescope such as the contemporary Hubbard optical instrument, visible electromagnetic radiation can be detected from a source with v_z of the order of 99% of c, so that significant amplification of linear and nonlinear induced polarization and magnetization would be expected, along with significantly increased optical rotations, according to equation (13). The total optical strength of these rotations would be proportional to the product of the total electric dipole moment (series (2)) and total molecular magnetic dipole moment (series (3)) of a suitable chiral material. To maximize the effect, a material would be chosen with an optical rotation to a sodium D-line, for example, of some thousands of degrees per metre. Visible radiation from the source must be gathered by the telescope, plane polarized, passed through the chiral material (a single crystal) and examined with an analyser. Fourier-transform methods can be implemented to resolve the incoming radiation into its frequency components and to plot the optical rotation as a function of frequency. Comparison of this with the same spectrum obtained with a broad-band source in the rest frame shows the effect of relativity.

To observe parity-violating optical rotations, the chiral single crystal is replaced with a structurally achiral heavy-metal vapour† in which P-violating optical activity is observable with a source in the rest frame. This is considerably amplified, theoretically, by using a source of radiation translating away from the rest frame at about $0{\cdot}99c$. This amplification may make it possible to detect molecular P-violating optical rotations too weak for observation with a source in the rest frame.

The Faraday effect, in which optical rotation is induced with a magnetic field in the rest frame, also becomes v dependent when the source recedes at a velocity near c. In this case, the interaction Hamiltonian (1) contains an extra term proportional to the earthbound applied magnetic field. Similarly, the magneto-chiral effect can be induced [18] with collinear magnetic and weak unpolarized electromagnetic fields, the former in the rest frame and the latter from the source in the far galaxy. Finally, spin chiral dichroism [23] also becomes v_z dependent, and can be induced with a strong circularly-polarized laser field in the rest frame colinear with weak unpolarized radiation from the source frame.

P-violating terms in the Hamiltonian in material present in the *source* frame between the source itself and the observe in the rest frame can be envisaged. This material could be the outer mantle of a sun. Electromagnetic radiation from the core passes through this before reaching the rest frame of the observer. In this case Lorentz corrections apply both to the electromagnetic radiation itself and to the P-violating term in the Hamiltonian. For weak current electron–nuclear interactions the latter is proportional to the vector product of the electron spin and electron linear momentum, both being subject to the Lorentz transformation. The net result is that this P-violating energy in the Hamiltonian is divided by the factor $(1-v^2/c^2)^{1/2}$ in the rest frame, and becomes theoretically infinite when $v_z=c$. P-violating terms in achiral material on an object receding from the Earth at a velocity near c are therefore greatly amplified by the Lorentz transformation.

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† See, for example, [6].

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