

A NEW METHOD FOR MEASURING THE DIELECTRIC LOSS OF HIGHLY ABSORBING MEDIA:  
VARIABLE ANGLE REFLECTIVITY FROM SURFACE LIQUID FILMS

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ABSTRACT

A method is developed for measuring the dielectric properties of lossy liquids by observing the power reflectivity in pi polarisation from a thin liquid film on aluminium substrate. The equations governing the interaction of microwave and radio frequency radiation with the liquid substrate system are solved using the inductance method of Hild and co-workers. This method takes advantage of the dependence of the power reflection coefficient on the film depth and angle of incidence of the incoming radiation and provides particularly interesting results near the Brewster angle.

INTRODUCTION

This letter introduces a new technique of microwave and radio frequency spectroscopy based on the reflectivity of electromagnetic radiation from a liquid film on metal substrate, in this case aluminium. The equations governing the interaction of microwave and radio frequency radiation with the surface film are solved using the inductance technique developed by Hild and co-workers [1-4]. A model system is set up whereby radiation at these frequencies is incident at any angle on the surface liquid film. By calculating the expected frequency dependence of the reflected radiation for a given dielectric loss it is shown that the power reflection coefficients ( $R_{\sigma}$  and  $R_{\pi}$ ) in sigma and pi polarisation are dependent on the angle of incidence  $\phi$ , and on the surface liquid film depth. This theoretical evaluation leads to a potentially useful method of investigating the dielectric properties of intensely absorbing media which present practical difficulties to conventional waveguide, bridge or sweep frequency techniques. [5]

## THEORETICAL BACKGROUND [1-3]

The electric and magnetic field vectors  $\vec{E}$  and  $\vec{H}$  of angular frequency  $\omega$  obey the following differential equations in an inhomogeneous medium of relative permittivity  $\hat{\epsilon}$  (usually a complex function), and a relative permeability  $\mu = 1$ .

$$\Delta \vec{E} + (\omega^2/c^2)\hat{\epsilon}\vec{E} - \text{grad div } \vec{E} = \vec{Q} \quad (1)$$

$$\Delta \vec{H} + (\omega^2/c^2)\hat{\epsilon}\vec{H} + (1/\epsilon)(\text{grad } \hat{\epsilon} \times \text{rot } \vec{H}) = \vec{Q} \quad (2)$$

It is assumed that the system is infinite in direction  $x$  and  $y$  and inhomogeneous only in axis  $z$ :

$$\hat{\epsilon} = \hat{\epsilon}(z) \quad (3)$$

The interface between the surface liquid film and the aluminium substrate is parallel to plane  $(x,y)$  and the liquid film is on a homogeneous infinitely thick substrate of pure aluminium. In this system the dependence of the field vector on  $z$  is treated separately for  $\sigma$  and  $\pi$  polarisation. In  $\pi$  polarisation the electric field is parallel to the plane of incidence and for  $\sigma$  polarisation perpendicular. The  $\sigma$  polarisation corresponds to the transverse electric (TE) mode, with  $\vec{E}$  in direction  $y$  and the  $\pi$  polarisation to the transverse magnetic (TM) with  $\vec{H}$  in direction  $y$ . The transverse field components  $E_y$  and  $H_y$  then obey the following differential equations:

$$\frac{\partial^2 E_y}{\partial z^2} + \frac{\omega^2}{c^2} (\hat{\epsilon} - \sin^2 \phi_0) E_y = 0 \quad (4)$$

$$\frac{\partial}{\partial z} \left( \frac{1}{\hat{\epsilon}} \frac{\partial H_y}{\partial z} \right) + \frac{\omega^2}{c^2} \left( 1 - \frac{\sin^2 \phi_0}{\hat{\epsilon}} \right) H_y = 0 \quad (5)$$

If now the admittance function is defined as:

$$\hat{j}(z) = - \frac{\mu_0}{\epsilon_0} \frac{1}{2} \frac{H_t(z)}{E_t(z)}, \quad (6)$$

then according to the boundary conditions of Maxwell's equations  $H_t$  and  $E_t$ , the transverse components, are continuous so  $\hat{j}(z)$  is a continuous function of  $z$  in the system, unless  $E_t = 0$  when the admittance function becomes infinite. The interaction of radiation with the inhomogeneous system can then be described with the following differential equations in the admittance function:

$$\frac{d\hat{j}_{TE}}{dz} = - \frac{i\omega}{c} \left[ \hat{\epsilon} - \sin^2\phi_0 - \hat{j}_{TE}^2 \right] \quad (7)$$

$$\frac{d\hat{j}_{TM}}{dz} = - \frac{i\omega}{c} \left[ \left( 1 - \frac{\sin^2\phi_0}{\hat{\epsilon}} \right) \hat{j}_{TM}^2 - \hat{\epsilon} \right] \quad (8)$$

These equations can be solved with the methods developed by Hild and co workers [1-3] for epitaxial semiconductors. Hild and Evans [3] have shown recently that particularly interesting effects can be observed in the region of the Brewster angle, defined [4] by:

$$\phi_B = \tan^{-1} \left[ |\epsilon_s^*|^{\frac{1}{2}} \right] \quad (9)$$

where  $\epsilon_s^*$  is the complex permittivity of the substrate, in this case Al. If radiation is incident on the surface liquid layer at or near the Brewster angle the reflectivity spectrum in pi polarisation becomes very sensitive both to slight changes in the angle of incidence and to slight changes in the depth of the surface liquid film. It can be shown as this letter that the reflectivity spectrum in the microwave and radio frequency region changes shape completely as the Brewster angle is transversed and this is of practical use for the dielectric spectroscopy of highly absorbing liquids.

#### OPTICAL COEFFICIENTS FOR THE ALUMINIUM SUBSTRATE

The optical properties of metallic aluminium have recently been investigated thoroughly [6] from the far infra red to the ultra violet, and it can be estimated that the permittivity of metallic aluminium up to about  $200 \text{ cm}^{-1}$  is approximately constant at 1.5 and that the dielectric loss is approximately constant at about 320,000. For a constant permittivity and dielectric loss the Brewster angle is also constant, and for aluminium substrate is dominated by the imaginary component:

$$\phi_B = \tan^{-1} 320,000^{\frac{1}{2}} = 89.8987^\circ \quad (10)$$

This means that in pi polarisation the reflectivity spectrum becomes extremely sensitive to angle of incidence a few minutes of arc above the parallel. With a beam directed at the surface liquid film at this glancing angle it is possible to measure the dielectric properties of very lossy liquids or materials deposited as thin films on the surface of the aluminium.

## RESULTS AND DISCUSSION

Figs (1) and (2) illustrates the sensitivity of the power reflectivity coefficient in pi polarisation ( $R_{\pi}$ ) to a change in incidence angle from  $\phi = 85^{\circ}$  to the Brewster angle of  $89.8987^{\circ}$ . The dielectric loss of the surface liquid film is the dashed curve (right hand ordinate) of fig. (1) to (5) of that figure refer to the R coefficient for a surface film depth decreasing from 1.0 mm to  $10^{-4}$  mm. For curve (1) interference fringes between the surface and the aluminium interface are just visible at high frequencies on the log scale of the abscissa. In great contrast the  $R_{\sigma}$  in fig. (1) is unity for all surface liquid depths and frequencies of incoming radiation. Therefore in this technique it is essential to use both pi and sigma polarisation. Note that the reflectivity coefficients of fig. (1) are generated mathematically from the dielectric loss curve by solving the fundamental equations for the system by the methods of Hild and co-workers [1-3]. The reflectivity coefficient characterises the dielectric property of the film substrate system and can be used to characterise that of the surface liquid knowing that of the Al substrate as in this letter.

Fig. (2) illustrates the dramatic change in the  $R_{\pi}$  spectrum as the incidence angle is changed to  $85^{\circ}$ . At lower frequencies the  $R_{\pi}$  coefficient shifts upscale from 0.2 to 0.95 for all liquid film depths. At higher frequencies (into the far infra-red) the onset of interference fringes is clearly visible and the reflectivity spectrum at these frequencies is sensitive to liquid film depth and angle of incidence. The five curves in fig. (2) correspond in film depth to the five curves in fig. (1) but are changed completely in frequency dependence. The method of reflectivity as described here therefore provides much more information by incorporating the dependence of the  $R_{\pi}$  spectrum on film depth and angle of incidence. Therefore it seems that the technique could provide a useful analytical method of investigating the dielectric properties of highly absorbing liquids.

Figs. (3a) and (3b) illustrate the change in the reflectivity spectrum when the incidence angle is reduced to  $60^{\circ}$ . The curves given in this figure are for four liquid thicknesses. For example, for 1 mm, high frequency fringes are clearly visible after a sharp fall in reflectivity in pi polarisation from near full scale to near zero. In contrast to fig. (1) the behaviour in sigma polarisation is similar to that in pi polarisation. At normal incidence the two reflectivity curves would be identical.

Finally, fig. (4) illustrates the reflectivity curves in pi polarisation for the same liquid thicknesses as in fig. (1). The dielectric loss curve in fig. (4) peaks however at far infra red frequencies so that the detail in the

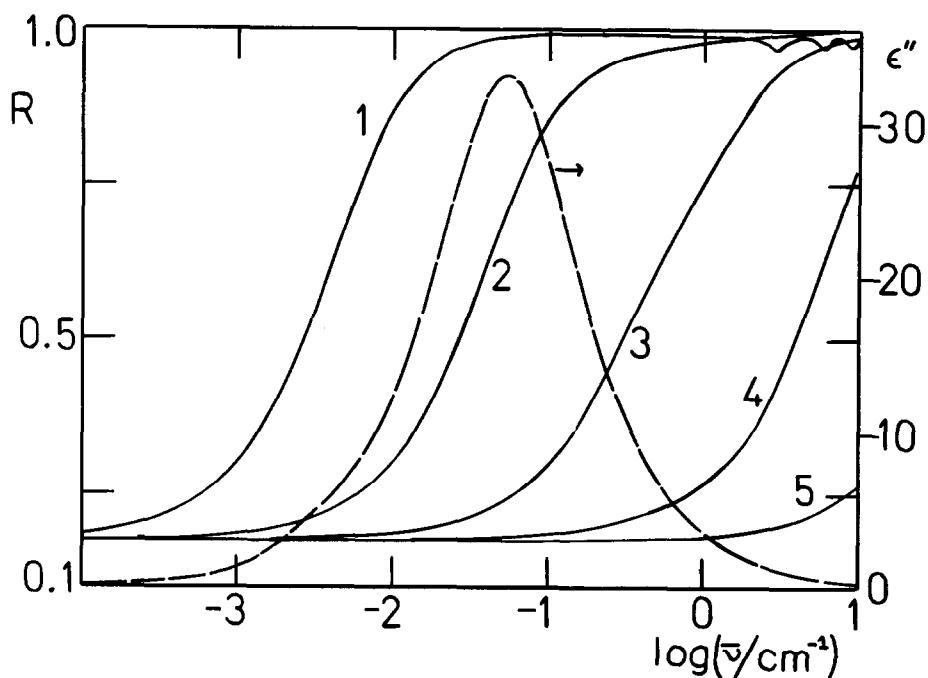


Fig. 1. Reflectivity in pi polarisation as a function of  $\log_{10}(\nu/\text{cm}^{-1})$ . Incidence angle  $\phi = 89.8987^\circ$ . 1. Surface film depth: 0.1 cm; 2.  $10^{-2}$  cm; 3.  $10^{-3}$  cm; 4.  $10^{-4}$  cm; 5.  $10^{-5}$  cm.

----- Dielectric Loss

Left Hand Ordinate:  $R_\pi$  : Right Hand Ordinate: Dielectric Loss.

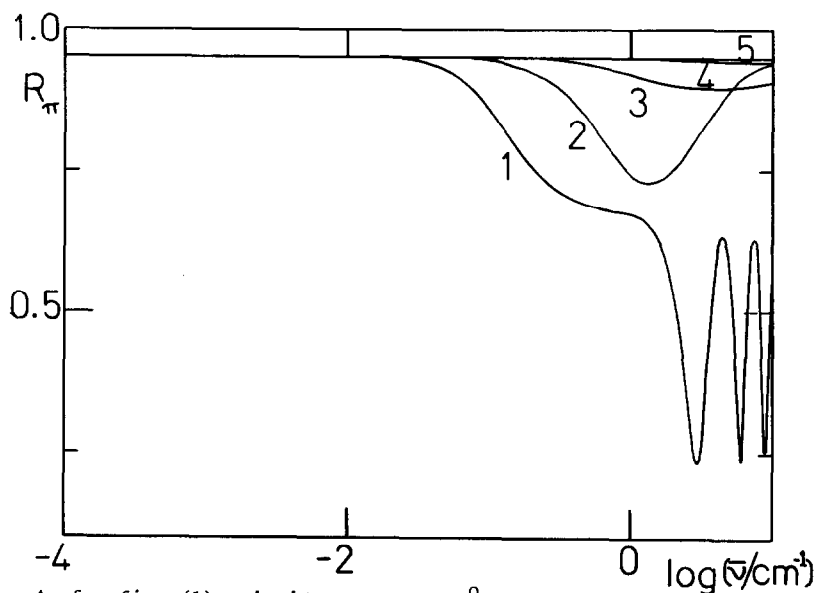


Fig. 2. As for fig. (1), incidence angle  $85^\circ$

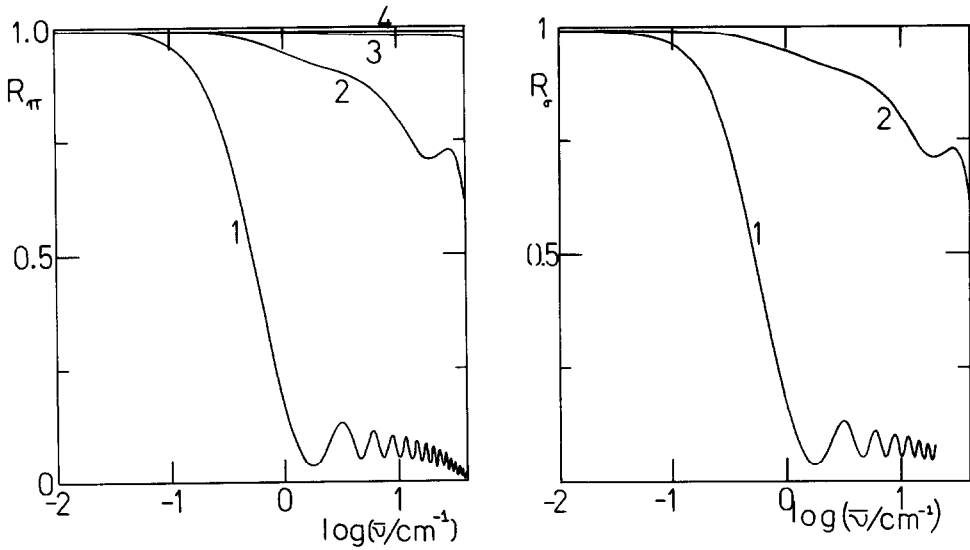


Fig. 3. Power reflectivity coefficients in pi polarisation. Incidence angle of  $60^\circ$ . 1. 0.1 cm; 2. 0.01 cm; 3.  $10^{-3}$  cm; 4.  $10^{-5}$  cm.

b) As for a) sigma polarisation.

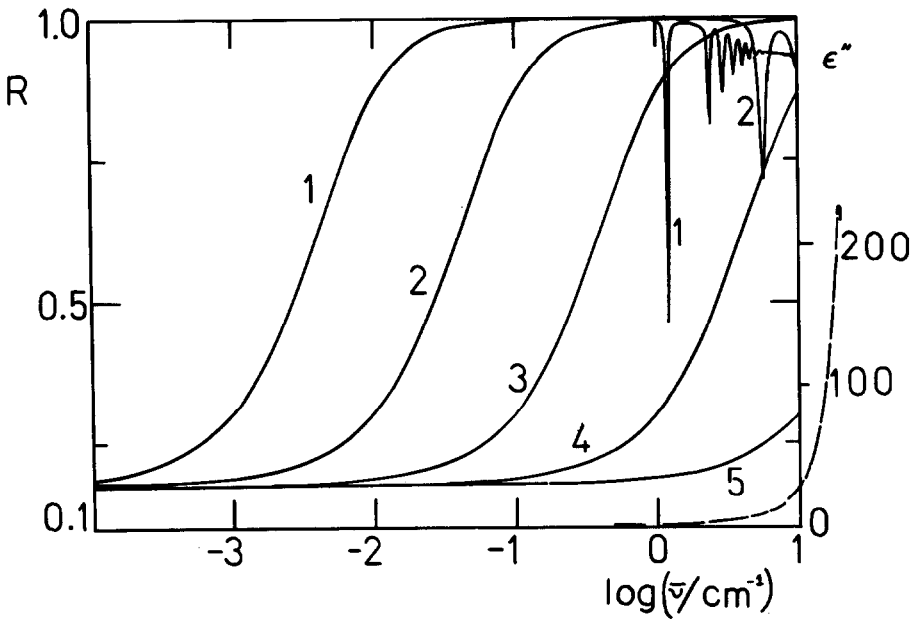


Fig. 4. As for fig. (1); dielectric loss curve in the far infra red region (dashed line).

power reflection coefficient in pi polarisation differs at high frequencies. It is interesting to note, however, that the power reflection coefficient at low frequencies continues to show detail which is dependent on the thickness of the surface liquid film in a way which is similar but not identical to the theoretical results in fig. (1). The thicker the film the lower the frequency at which the "band edge" occurs in pi polarisation at the Brewster angle.

#### CONCLUSIONS

By investigating the frequency dependence of the reflectivity at microwave and radio frequencies from a surface liquid film deposited on an Al substrate a new technique is suggested for investigating the dielectric properties of liquids whose dielectric loss is too great for investigation with conventional techniques. The reflectivity spectra seem to be particularly sensitive to incidence angle of radiation near the Brewster angle in pi polarisation, and advantage of this could be taken experimentally.

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